P-31

3D FDTD-PIC Simulation of Particles in a Miniature Ion Thruster

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Abstract

Our research focuses on simulation of particles inside an Electric Ion thrusters. An Ion engine works by creating Plasma through ECR (Electron Cyclotron Resonance) method and accelerating the resultant ions out of the engine thus producing thrust. It is, however, difficult to measure inner properties of the plasma, since the ion engine is so small that we cannot insert measurement equipment without perturbations. Numerical simulations are effective tool to understand the phenomena occurring in a miniature microwave ion thruster. This research focuses on development of a three dimensional FDTD (Finite Difference Time Domain) approximation to Maxwell's equations simulation for propagation of TEM (Transverse Electro-Magnetic) microwaves in a Coaxial Cable and development of PIC (Particle In Cell) simulation for particle interactions inside the miniature ion thruster.

1. Introduction

Current research involves the development of an all-inclusive full PIC-FDTD code in order to simulate the particles inside a miniature ion thruster. As mentioned earlier it is very challenging to measure inner properties of the plasma inside the very small thruster, a fully developed Numerical simulations code which utilizes the PIC method as well as FDTD would shed light on the inner workings of our engines. FDTD (Finite Difference Time Domain) simulation method is extensively used in plasma simulation as a way to calculate the fluctuations of electric and magnetic field values with respect to space and time. Therefor this method is utilized to simulate propagation of electromagnetic waves and charged particles. Coaxial cables are used in order to feed the microwave power to the engine and an accurate simulation of microwave propagation inside a coaxial cable leads to a more realistic simulation of particles for the entire engine. Figure 1 shows one such engine developed in our research group in Kyushu University with the coaxial cable attached to the thruster. Our current research involves simulation of microwaves in the coaxial cable as well as simulation of neutral particles in the engine.

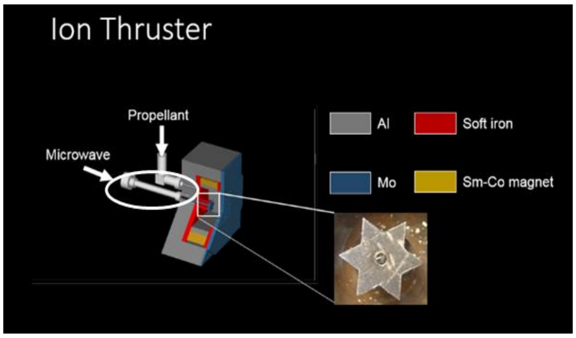

Fig. 1: A miniature Ion thruster developed in Yamamoto lab In Kyushu University

2. FDTD simulation

2.1 Electromagnetic Fields

Since the three dimensional Cartesian model is used for this simulation, each grid is practically a perfect cube. One such grid along with the location of the field points is illustrated in Figure 4. Since none of the field points fits on the grid, a 3D FDTD simulation is very challenging in its calculation and indexing. Adjusting the original Maxwell formulas to the simulation model in Figure 4 is a challenging task. Considering the basic scalar Maxwell equations in three dimensions:

$$\frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \qquad eq.1$$

$$\varepsilon \frac{\partial E_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) \qquad eq.2$$

By transforming these equations to the grid point system in Figure 2, Maxwell's equations can be utilized in the simulation. Equations 3 and 4 indicate such transformation for E_x and B_x:

$$\varepsilon_{di} \frac{E^{n}_{x\left(i+\frac{1}{2},j,k\right)} - E^{n-1}_{x\left(i+\frac{1}{2},j,k\right)}}{dt} = \frac{1}{\mu_0}\ \left(\frac{B^{n-\frac{1}{2}}_{z\left(i+\frac{1}{2},j+\frac{1}{2},k\right)} - B^{n-\frac{1}{2}}_{z\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}}{dy} - \frac{B^{n-\frac{1}{2}}_{y\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)} - B^{n-\frac{1}{2}}_{y\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}}{dz} \right) \qquad eq.3$$

$$\frac{B^{n+\frac{1}{2}}_{x\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)} - B^{n-\frac{1}{2}}_{x\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)}}{dt} = \frac{E^{n}_{y\left(i,j+\frac{1}{2},k+1\right)} - E^{n}_{y\left(i,j+\frac{1}{2},k\right)}}{dz} - \frac{E^{n}_{z\left(i,j+1,k+\frac{1}{2}\right)} - E^{n}_{z\left(i,j,k+\frac{1}{2}\right)}}{dy} \qquad eq.4$$

And finally by adjusting the indexing of the above equations for the simulation code we will have:

$$E_{x(i,j,k)} = E_{x(i,j,k)} + \left(B_{z(i,j,k)} - B_{z(i,j-1,k)}\right)C_y - \left(B_{y(i,j,k)} - B_{y(i,j,k-1)}\right)C_z \qquad eq.5$$

$$B_{x(i,j,k)} = B_{x(i,j,k)} + \left(E_{y(i,j,k+1)} - E_{y(i,j,k)}\right)\frac{1}{2}\frac{dt}{dz} - \left(E_{z(i,j+1,k)} - E_{z(i,j,k)}\right)\frac{1}{2}\frac{dt}{dy} \qquad eq.6$$

Where $C_y = \frac{1}{2}\frac{dt}{dy}\frac{1}{\varepsilon_{di}\mu_0}$ and $C_z = \frac{1}{2}\frac{dt}{dz}\frac{1}{\varepsilon_{di}\mu_0}$.

2.2 Microwave Input Method

The microwave input power in a coaxial cable is calculated by:

$$power = \frac{2\pi}{\sqrt{\frac{\mu_0}{\varepsilon_{di}}}} v_{0b}^2 \ln\left(\frac{r_{outer}}{r_{inner}}\right) \qquad eq.7$$

Where r_{outer} and r_{inner} are the outer and inner radius of the cable respectively and the v_{0b} is the voltage between the cable's inner and outer layers. Therefore input field values are calculated by:

$$B_\theta = v_{0b}\frac{1}{r}\frac{1}{c_D} \quad , \qquad E_r = v_{0b}\frac{1}{r} \qquad equ.8$$

Where Cd is the speed of light in the dielectric region. The field components E_r and B_θ should be implemented on the cables cross section (Figure 3) and should be divided into their xy components. In order to get the right $E \times B$ vector and the right values, therefore the following formulation is used:

$$E_{x(i,j,k)} = E_{x(i,j,k)} + E_{r(i,j)}\frac{x}{r} \qquad equ.9$$

$$E_{y(i,j,k)} = E_{y(i,j,k)} + E_{r(i,j)}\frac{y}{r} \qquad equ.10$$

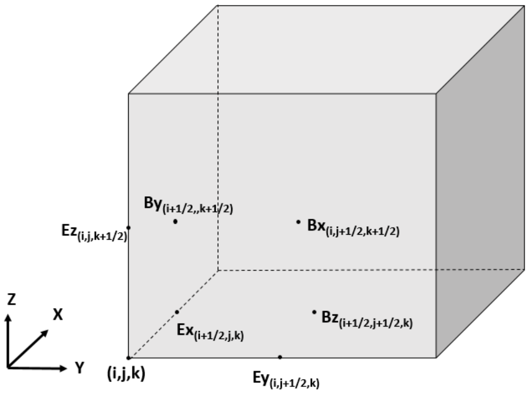

Fig. 2: a 3D FDTD grid with field positions

$$B_{x(i,j,k)} = B_{x(i,j,k)} - B_{\theta(i,j)}\frac{y}{r} \qquad equ.11$$

$$B_{y(i,j,k)} = B_{y(i,j,k)} + B_{\theta(i,j)}\frac{x}{r} \qquad equ.12$$

The simulated waves are illustrated in figures 3 through 8. The results in these figures are taken at a mid-plane cross section of the cable where the y values are constant. Since the wave in this simulation is supposed to act as a TEM wave the E_z and $B_z$values are expected to be close to zero. As it can be seen in figure 5 the comparative value of E_z with respect to the amplitude of E_x wave in figure 3 is too insignificant (24000 times smaller) which means that it can be ignored. Same thing applies to the magnetic field values depicted in figures 7 and 8. The B_z value is half a million times smaller than $B_y$which means it will not affect the accuracy of the simulation as well. Since this output data is taken at a plane in the center of the y axis, E_y and $B_x$values are also expected to be zero. However as mentioned earlier since the simulated geometry of the cable is not a perfect cylinder we expect some imprecisions as well. According to figures 4 and 6 since the amplitudes of E_y and B_x waves are four orders of magnitude smaller than their respective wave amplitudes, these imperfections can also be ignored as they will not cause any defects on the outcome of the simulation.

P-31

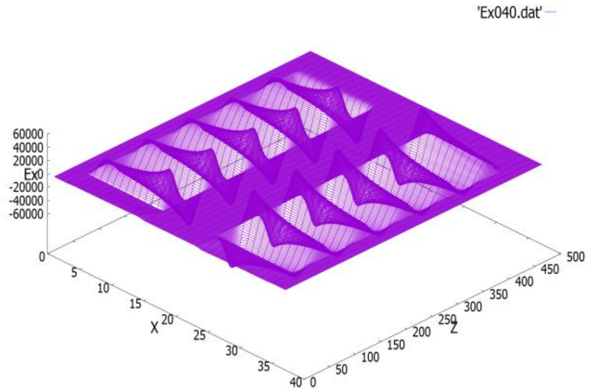

Fig. 3: E_x field values through the coaxial cable

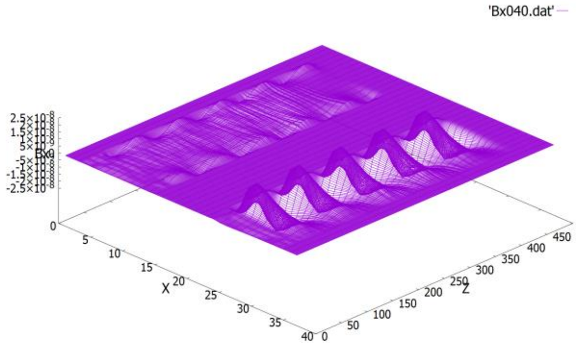

Fig. 6: B_x field values through the coaxial cable

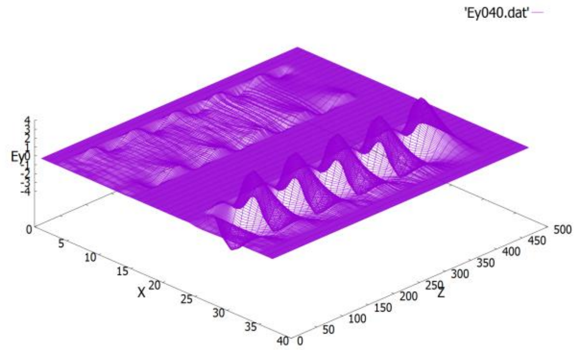

Fig. 4: E_y field values through the coaxial cable

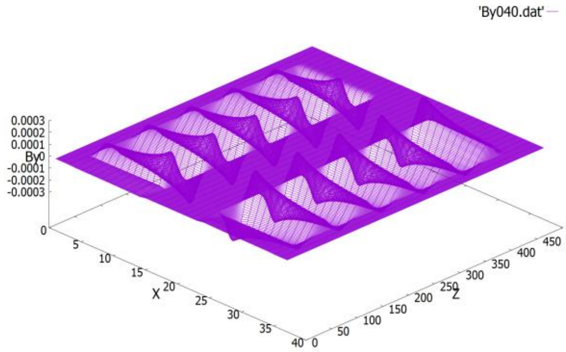

Fig. 7: B_y field values through the coaxial cable

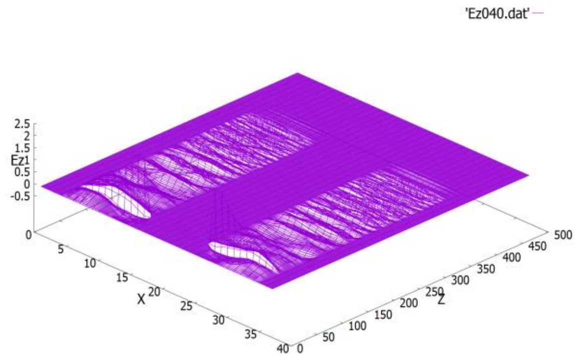

Fig. 5: E_z field values through the coaxial cable

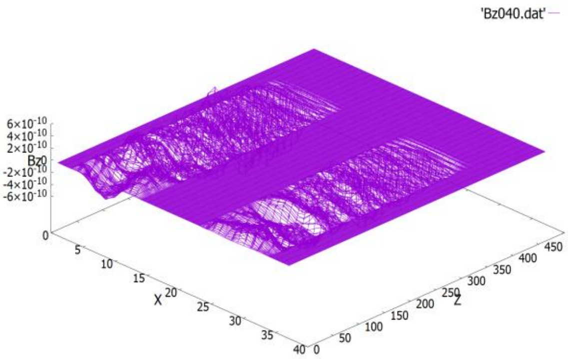

Fig. 8: B_z field values through the coaxial cable

3. PIC simulation

For the analysis of plasma behavior in the discharge chamber, the PIC method is used. Effects of electromagnetic field on plasma particles are calculated by linear interpolation in the PIC method. The PIC method can treat collision process and distribution function since in this method the simulation space is divided into meshes in which individual particles are tracked along these meshes while variables such as density and current are calculated at these mesh points. Using this method the particle interaction with the engine and each other such as reflection on the engine walls and collision of particles is also simulated. In order to achieve that, first we have simulated the interaction of neutral particles in the engine. Figures 9 shows the particles distribution of neutral particles in a 3D engine. In order to simplify the outcome demonstration, these results are taken at a central plane rather than the whole engine.

4. Conclusion

The simulation for a microwave propagation inside a coaxial cable was discussed using FDTD method in a 3D model. The simulation results were discussed and their validation was investigated. Furthermore the simulation results of a 3D PIC simulation of neutral particles in miniature ion thruster was discussed. These two codes will be utilized in future works for a more realistic simulation of an Ion thruster. This new simulation model will replace the previous inaccurate method for future simulations in our research group and This in turn will result in development of better performing engines in future.

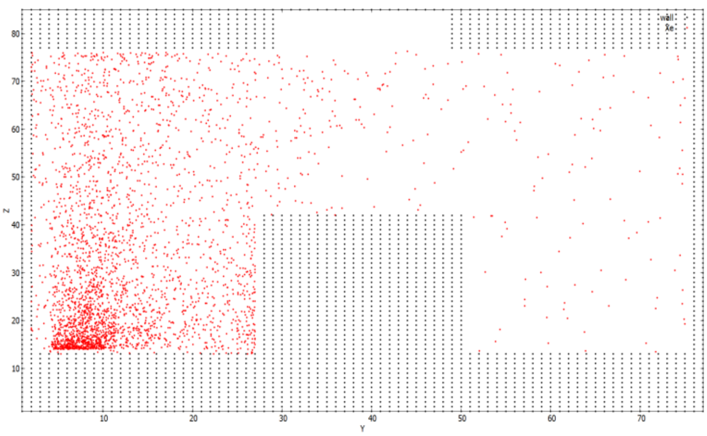

Fig. 9: Neutral particle distribution in a miniature Ion thruster developed in Yamamoto lab In Kyushu University

Reference

[1]. Francis F.Chen, in Introduction to Plasma Physics and Controlled Fusion, 2nd ed. by Plenum Publishing Corporation, (1974).

[2]. C.K.Birdsall and A.B.Langdon, in Plasma Physics via Computer Simulation, by IoP Publishing (1985).

[3]. Yee K 1966 “Numerical solution of initial boundary valueproblems involving maxwell’s equations in isotropic media” IEEE Trans. Antennas Propagat.

[4]. Kousaka.H, Ono.K 2002 “ Numerical Analysis of the electromagnetic fields in a Microwave Plasma Source Excited by Azimuthally Surface Waves” Jpn J. Phys. Vol 41

[5]. Takao Y, Koizumy H, Eriguchi K, Komurasaki K, and Ono K 2012 “Three-dimensional Particle in Cell Simulation of Miniature Plasma Source for a Microwave Discharge Ion Thruster ” Plasma Source Sci. Technol Vol.23

[6]. Koizumi H and Kuninaka H 2010 “Miniature microwave discharge ion thruster driven by 1 watt microwave power” J. Propul. Power 26 601-4

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