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Directional reversal of solitary wave in Ising type Vicsek model

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Abstract: *We study collective phenomena of self-propelled particles using one-dimensional Ising type Vicsek model by direct numerical simulation. The velocity takes 1 or -1 and multiple particles can occupy one lattice point. We find direction of propagating solitary wave consisted of them reverses stochastically. The peak of the solitary wave is highest immediately after reversal; The wave becomes smaller and wider with time. The reversal phenomenon occurs by colliding broad wave and adverse wave of very small amplitude.*

Keywords: Self-propelled particle; Collective motion; Vicsek model; Ising model; Solitary wave.

1. INTRODUCTION

The collective motions of self-propelled particles such as school of fish and flocks of birds are found in nature. They have been intensively studied since Vicsek et al. proposed simple model [1]. Various agent-based models similar to the Vicsek model have been proposed and numerically studied to investigate the dynamical behaviors of a large population of self-propelled particles [2]. Collective motion of biological systems was found experimentally and has been studied [3, 4]. In these experiments, large structures were formed out of small traveling objects.

In the Vicsek model (similarly VM hereafter), self-propelled particles with a constant absolute velocity interact with other particles. The particles can go in any direction in plane. At next time step, the particle goes to the average direction of motion of other particles within a distance from the particle. In addition, a noise is added for the average direction. In Vicsek-type models, order–disorder transition is seen. When the noise effect is dominant, the particles move randomly, that is the disorder state. When the interaction for alignment dominates, the particles move almost in the same direction, that is the order state. In addition, it was found propagating localized regions of high density with a constant velocity similarly to solitary waves [5]. This phenomenon appears near transition line. The spatial distribution of the solitary wave state in two-dimensional system is nonuniform for one axis, but uniform for the other. Here, disorder state is appeared as the background. The solitary wave state appeared in one-dimensional system [6]. The solitary wave state appears away from transition line. This phenomenon was arisen owing to an instability from spatial uniform state in the nonlinear Kramers eqation. In addition, A reversal of localized cluster of self-propelled particles appears in active Ising model [7].

2. ISING TYPE VICSEK MODEL

Ising model is famous as a model of phase transition of ferromagnet. The state in Ising model have either upward or downward spin on lattice. The spin on the lattice interacts with its nearest neighbors and changes the spin direction at next time step. Here, we propose Ising type Vicsek model to study behavior of collective motion of self-propelled particles. Multiple particles which are moving right or left can stay on a lattice point in this model as shown in Fig. 1(a). The velocity of particle is 1 or -1 on one-dimensional lattice and the particle interacts with other particles on the lattice point and its neighboring points. At next time step, each particle travels in the direction determined by majority rule of particles on the lattice points with high probability p. Then, p is probability providing $p = e^{g}/(e^{g} + e^{-g})$, where g is parameter. At $g = 0$, particles move randomly without aligning. For the majority rule of the direction, a parameter Q is used.

$$Q_i = P_i - M_i + 1/2(P_{i-1} - M_{i-1}) + 1/2(P_{i+1} - M_{i+1}), \quad (1)$$

where P_i and M_i are number of particles moving to right

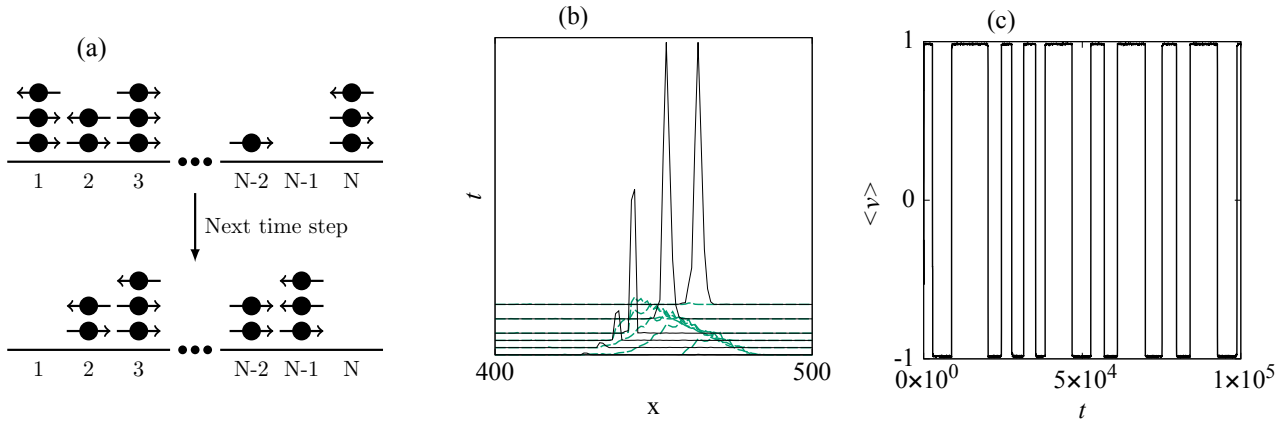

Fig. 1. (a) Ising type Vicsek model. (b) Reversal of solitary wave. (c) Average velocity of all particles.

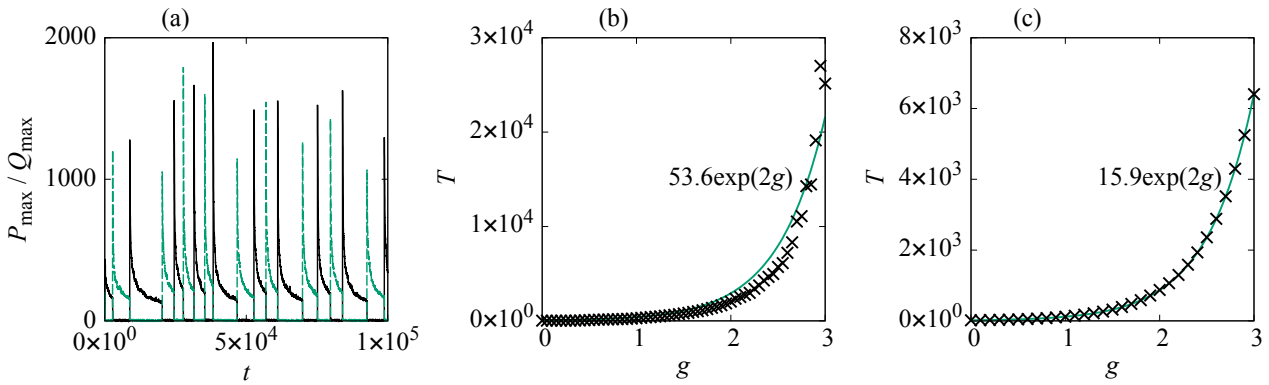

Fig. 2. (a) Maximum number of particles moving to right or left. (b) Average reversal time in this model. (c) Decay time in Eq. (2).

and left of ith element. All particles N obey above the rule on the one-dimensional lattice L applied periodic boundary condition. The Monte Carlo simulation is performed from disorder state as initial condition.

With time, if probability is high, groups of particles propagating to the same direction are formed out of the disorder state where particles are randomly moving. The groups repeat reversal and form a solitary wave of which peak is highest immediately after reversal. Figure 1(b) shows the time evolution of the maximum number of particles moving to right or left at $N = 5000$, $L = 500$, $g = 2.5$. The wave (dashed line) which is smaller amplitude and wider with time reverses by colliding with adverse wave (solid line) which is very small amplitude. The same parameter values are used for Fig. 1(b), (c), and Fig. 2(a). Figure 1(c) shows time series of the average velocity of all particles $\langle v_i \rangle = (1/N)\sum v_i$. At $< v > = 0$, particles move randomly. At $< v > = 1$, all particles move to right. The reversal of solitary wave occurs rapidly and the interval between two reversals is not constant. Figure 2(a) shows time series of maximum number of particles moving to right or left, which are denoted by solid and dashed line, respectively. At first, the peaks are attenuated quickly, and the decrements are gradually slow for later time. The peaks correspond to reversals of the average velocity between -1 and 1 in Fig. 1(c).

Here, we think the movements of particles at next step in this model. The number of particles moving to right P_i^{n+1} and left M_i^{n+1} are expressed as

$$P_i^{n+1} = pP_{i-1}^{n} + (1-p)M_{i-1}^{n}, \quad (2a)$$

$$M_i^{n+1} = (1-p)P_{i+1}^{n} + pM_{i+1}^{n}, \quad (2b)$$

where p is constant despite of the majority rule. If the probability p is high and the solitary wave state for right is strong, by using the condition: $M_i^{n} \sim 0$, Eq. (2a) is satisfied

$$P_i^{n+1} = pP_{i-1}^{n}. \quad (3)$$

The peak is decreased like p^n with time n. If the reversal occurs at $\alpha < 1$, the following relation is given as

$$p^n = \alpha. \quad (4a)$$

$$n = \frac{\ln \alpha}{\ln p} = \frac{\ln \alpha}{\ln\{e^g/(e^g+e^{-g})\}} = -\frac{\ln \alpha}{\ln(1+e^{-2g})} \sim (-\ln \alpha)e^{2g}. \quad (4b)$$

Figure 2(b) shows average reversal time T in this model at $N = 5000$, $L = 500$. The cross points are numerical results and solid line is the fitting curve, which is $T(g) = 53.6\exp(2g)$. This numerical result agrees well with the approximation in Fig. (4b). For $g > 2.6$, there is deviation for the average owing to short of number of reversals. Figure 2(c) shows a decay time T to reach $1/10P_{max}$ in Eq. (2) at $N = 2000$, $L = 500$, $P_{L/2} = P_{L/2+1} = 1000$. The fitting curve is $T(g) = 15.9\exp(2g)$. This result is close to the result in Fig. 2(b) in Ising type Vicsek model.

3. SUMMARY

We have studied the Ising type Vicsek model for one-dimensional self-propelled particles. The direction of propagating solitary wave reverses stochastically. The peak and the reversal time are not constant. The reversal time is approximated to e^{2g}. We would like to generalize this model, taking effect of broader interaction or in two-dimensional system for further understanding the phenomenon.

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