Proceedings of International Forum for Green Asia 2019
5th International Exchange and Innovation Conference on Engineering & Sciences (IEICES 2019)

GA08 / P20

Normalization of gas transfer velocity at air-water interface

Sung Jin Kim, Yuji Sugihara
Interdisciplinary Graduate School of Engineering Sciences, Kyushu University
Email: user0galaxy@gmail.com

Abstract: *The turbulent motion that control the gas transfer velocity at the air-water interface is divided into two types of turbulent scales, which are integral scale and microscale such as the Taylor microscale and Kolmogorov scale. The gas transfer velocity can be expressed based on these turbulence scales. The turbulent scale to model the gas transfer velocity is expected to vary with Reynolds number. The purpose of this study is to carry out the normalization of the air-water gas transfer velocity using a new turbulent time scale combining the two time scales. Comparing this normalization with experimental data of oscillating grid turbulence shows that this result helps us describe the gas transfer velocity over a wide range of Reynolds number.*

Keywords: turbulence, gas transfer velocity, air-water interface, combined turbulent time scale

1. INTRODUCTION

The gas flux on the air-water interface is generally expressed in the form of the bulk expression using the gas transfer velocity and the difference of dissolved gas concentration. Based on the surface renewal theory, the gas transfer velocity is expressed by using a turbulent time scale, replacing the renewal rate, as follows:

$$k_L = C\sqrt{\frac{D}{\tau}} \tag{1}$$

where D, τ and C are the diffusion coefficient, turbulent time scale and a coefficient of proportionality. Therefore, the formulation for the gas transfer velocity varies depending on turbulent time scale in this equation (1). As the turbulent scale controlling the variation of gas transfer, there are two types of models well known, which are Large eddy model with the eddies at the integral length scale and Small eddy model with the eddies at micro scales[1].

The gas transfer velocity is described using each turbulent time scale and its dimensionless form becomes

$$\tau = \tau_L = \frac{k}{\varepsilon} \quad \rightarrow \quad \frac{k_L Sc^{1/2}}{k^{1/2}} \propto Re_t^{-1/2} \tag{2}$$

$$\tau = \tau_s = A\sqrt{\frac{\nu}{\varepsilon}} \quad \rightarrow \quad \frac{k_L Sc^{1/2}}{k^{1/2}} \propto Re_t^{-1/4} \tag{3}$$

where k, ε, ν, and A are the turbulent energy and the energy dissipation rate at the air-water interface, the kinematic viscosity coefficient on liquid side, and the coefficient of proportionality. Also, Sc denotes Schmidt number, and the turbulent Reynolds number Re_t is defined by

$$Re_t = \frac{k^2}{\varepsilon\nu} \tag{4}$$

From these dimensionless relations, we found the gas transfer velocity of LEM and SEM to show the scaling laws proportional to -1/2 or -1/4 to the turbulent Reynolds number, respectively. In addition to LEM and SEM, there is the surface divergence model (SDM) using the surface divergence calculated from horizontal flow velocities on the air-water surface. Tsumori and Sugihara[2] showed that SDM is equivalent to using Taylor micro scale as the turbulent scale, and that the scaling relation based on SDM is formally consistent with SEM in this case.

How should we judge whether LEM and SEM scaling rules are valid or not for turbulent fields we consider? One idea is to investigate how the dimensionless gas transfer velocity varies depending on the turbulent Reynolds number. Figure 1 shows the experimental results of oscillating grid turbulence provided by Mitani[3]. Here, the turbulent energy and energy dissipation rate at the air-water interface used for the normalization have been evaluated using the semi-empirical relations for the oscillating grid turbulence proposed by Matsunaga et al.[4]. The plots in the figure also display the experimental values. The red and blue lines shows the LEM relation (2) and the SEM relation (3), respectively. From this figure, the experimental values seem to agree with the LEM relation in the range of low Reynolds number, whereas the SEM relation appears in the high Reynolds number. This fact suggests that the turbulent scale controling the gas transfer changes with Reynolds number.

In this study, based on knowledge about these turbulent scales, we discuss the normalization of the gas transfer velocity at the air-water interface and the empirical expression for the gas transfer velocity applicable over a wider range of the Reynolds number.

2. Empirical expression of turbulent time scale and gas transfer velocity

Following the method of Yoshizawa, the combined time scale for multiple turbulent time scales may be shown as

$$\frac{1}{\tau^2} = \frac{1}{\tau_s^2} + C_L\frac{1}{\tau_L^2} \tag{5}$$

where C_L is an empirical constant. Therefore, the turbulent time scale τ gives the following relations:

$$\tau = \frac{\tau_s}{\sqrt{1 + C_L\left(\tau_s/\tau_L\right)^2}} = \frac{\tau_s}{\sqrt{1 + C_1 R_{et}^{-1}}} \tag{6}$$

The relations of equations (2) and (3) are used for τ_L and τ_s, and C_1 is a new empirical constant. In equation (5), the combined time scale is defined in the form of the sum of inverse squares, but we consider an empirical relation,

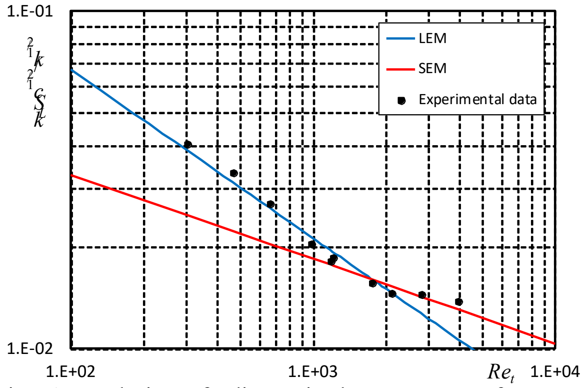

Fig. 1. Relation of dimensionless gas transfer velocity with turbulent Reynolds number.

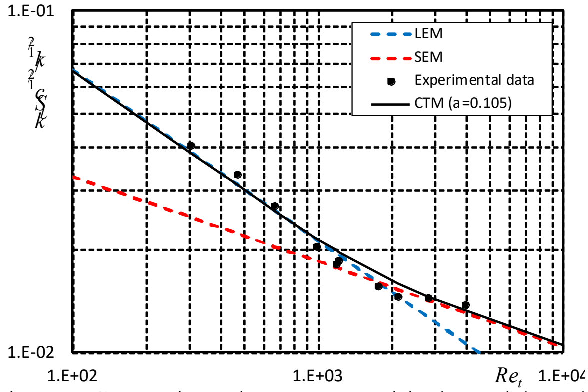

Fig. 2. Comparison between empirical model and experimental data.

so that it may not be necessary to express the turbulent time scale by the sum of inverse square. In order to more generally express the time scale under consideration of equation (6), we propose the relation as

$$\tau = \frac{\tau_s}{\left(1 + C_1 Re_t^{-n/2}\right)^{1/n}} \tag{7}$$

with a constant being n = 1, 2, 3,…, but it is a lack of physical meaning except for n = 1 and 2.

Substituting the combined time scale of equation (7) into equation (1) and deriving the gas transfer velocity, we obtain the following relation:

$$\frac{k_L S_c^{1/2}}{k^{1/2}} = \alpha\, Re_t^{-1/4} \left[1 + \left(\frac{Re_t}{Re_{tc}} \right)^{-n/2} \right]^{1/(2n)} \tag{8}$$

where α is an empirical constant, and Re_{tc} is the critical Reynolds number at which the turbulence scale changes. The above relation has been expressed based on SEM, and the selection of n = 2 corresponds to the solution of equation (6). Here, figure 2 shows the comparison between equation (8) in the case of n=6 and the experimental results shown in figure 1. From this figure, the dimensionless representation of equation (8) appears to fit over a wide range of the turbulent Reynolds number. We should note that as n increases, the kink near the critical Reynolds number becomes clearer and the degree of agreement between LEM and SEM increases, independent of the validity of the value of n.

3. Summary

The gas transfer velocity at the air-water interface has been modeled and normalized using an empirical combined time scale based on two turbulent time scales. Though the theoretical background is somewhat sparse, such an empirical expression can be expected to apply to the behavior of the gas transfer velocity over a wide range of the turbulent Reynolds number.

4. REFERENCES

[1] Sugihara, Y., Sanjou, M., "Turbulence and Gas Transfer in Interfacial Hydraulics -Progresses of Experiments and Medels-", *Nagare*, 30, 181-193, 2011. (In Japanese)

[2] Tsumori, H., Sugihara, Y., "Lengthscales of motions that control air-water gas transfer in grid-stirred turbulence", *J. Marine Systems*, 66, 6-18, 2007.

[3] Mitani, S., "Universal description of gas transfer mechanism at the air-water interface", Master's thesis, Dept. Earth System Science and Technology, Interdisciplinary Graduate School of Engineering Scieces, Kyushu university, 2009. (In Japanese)

[4] Matsunaga et al., "Quantitative properties of oscillating-grid turbulence in a homogeneous fluid, *Fluid Dynamics Res.*, 25, 147-165, 1999.

[5] Yoshizawa, A.， "Chapter 3 Turbulent Transports and Reynolds-Averaged Modeling", *Nagare*, 30, 241-261, 2011. (In Japanese)