Study on Plasma Shape Reproduction of Spherical Tokamak using CCS Method

Feng WANG [*1,†], Kazuo NAKAMURA [*2], Osamu MITARAI [*3], Kenichi KURIHARA [*4], Yoichi KAWAMATA [*4], Michiharu SUEOKA [*4], Kohnosuke SATO [*2], Hideki ZUSHI [*2], Kazuaki HANADA [*2], Mizuki SAKAMOTO [*2], Hiroshi IDEI [*2], Makoto HASEGAWA [*2], Shoji KAWASAKI [*2], Hisatoshi NAKASHIMA [*2], Aki HIGASHIJIMA [*2]

[†] E-mail of corresponding author: *maple@aees.kyushu-u.ac.jp*

(Received April 27, 2007)

Cauchy-Condition Surface (CCS) method is a numerical approach to reproduce plasma shape which has good precision in conventional tokamak. In order to apply it in the plasma shape reproduction of Compact PWI experimental Device (CPD) which is a new spherical tokamak in Kyushu University, the calculation results of CCS method on CPD is reported in the paper.

Key words: *Plasma, Spherical tokamak, CPD, Cauchy-Condition surface, Shape reproduction*

1. Introduction

From the viewpoint of plasma control of tokamak, plasma shape reproduction is very important, especially for the non-circular and triangular plasmas. Compared with the conventional tokamak, spherical tokamak has high natural elongation and natural triangularity due to smaller aspect ratio, and the equilibrium and stability properties are different. There are some kinds of numerical methods to reproduce plasma shape such as filament current method. The CCS method is a kind of numerical approach to reproduce plasma shape by using magnetic measurement which shows good results in comparison with filament current method and equilibrium method. It can reproduce plasma shape with precision corresponding to the number and types of available sensors, and it can be used in the real-time plasma shape control. In order to apply it in the plasma shape control of CPD which is a spherical tokamak, the calculation precision is studied.

2. CCS Method Outline

The Cauchy-Condition Surface method is a kind of exact numerical method which is based on the boundary integral equation. The Cauchy-Condition surface is defined as a hypothetical plasma surface, where both the Dirichlet (ϕ) and Neumann (Bt) conditions are unknown, as shown in Fig. 1. This surface is located inside the real plasma region. It is assumed that CCS encloses all the plasmas and there are no plasmas outside the CCS [1-2].

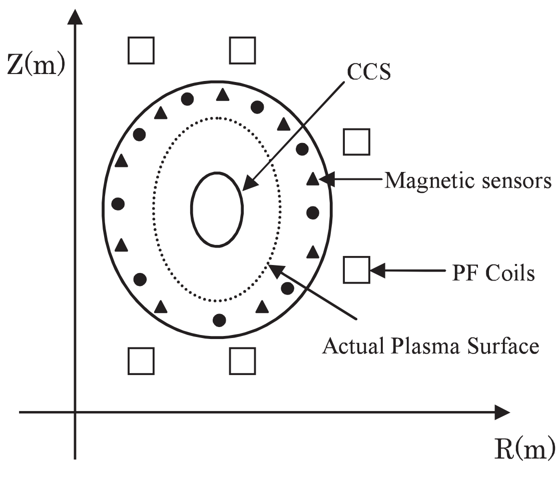


Fig. 1 Outline of Cauchy-Condition Surface method

According to the static Maxwell's equation, three types of boundary integral equations can be given by using the magnetic sensors signals and poloidal coils current data. The discretized formulas for flux loops, magnetic probes and

*1 Department of Advanced Energy Engineering Science
*2 RIAM, Kyushu University
*3 Kyushu Tokai University
*4 Japan Atomic Energy Agency

CCS are shown as follows [3].

$$\phi(x_f)=\sum_{i=1}^{M}W_{F1}(x_f,z_i)\phi(z_i)+\sum_{i=1}^{M}W_{B1}(x_f,z_i)\mathrm{B}t(z_i)+W_{C1}(x_f)I_{PF} \quad (1)$$

$$Bt(x_B)=\sum_{i=1}^{M}W_{F2}(x_B,z_i)\phi(z_i)+\sum_{i=1}^{M}W_{B2}(x_B,z_i)\mathrm{B}t(z_i)+W_{C2}(x_B)I_{PF} \quad (2)$$

$$\frac{1}{2}\phi(x)=\sum_{i=1}^{M}W_{F3}(x,z_i)\phi(z_i)+\sum_{i=1}^{M}W_{B3}(x,z_i)\mathrm{B}t(z_i)+W_{C3}(x)I_{PF} \quad (3)$$

Where, M is the number of discretized points along CCS. $\phi(x_f)$ is the flux loop measurement. $Bt(x_B)$ is the magnetic probe measurement. $\phi(z_i)$ and $\mathrm{B}t(z_i)$ is the flux and Bt value of discretized points on CCS. I_{PF} is the poloidal field coils current included in the calculation region.

$W_{F1}(x_f,z_i)$, $W_{B1}(x_f,z_i)$, $W_{C1}(x_f)$, $W_{F2}(x_B,z_i)$, $W_{B2}(x_B,z_i)$, $W_{C2}(x_B)$, $W_{F3}(x,z_i)$, $W_{B3}(x,z_i)$, $W_{C3}(x)$ are coefficient matrix which can be calculated beforehand.

Equations (1), (2) and (3) are coupled and can be expressed in matrix form, and then $\phi(z_i)$ and $\mathrm{B}t(z_i)$ at several discretized points along CCS can be evaluated by using the least square method.

Then the flux distribution can be calculated using equation (4), and the outmost magnetic flux surface or plasma shape can be found by plotting the contour.

$$\phi(x)=\sum_{i=1}^{M}W_{F4}(x,z_i)\phi(z_i)+\sum_{i=1}^{M}W_{B4}(x,z_i)\mathrm{B}t(z_i)+W_{C4}(x)I_{PF} \quad (4)$$

Where, $\phi(x)$ is the flux value at any position. $W_{F4}(x,z_i)$, $W_{B4}(x,z_i)$ and $W_{C4}(x)$ are coefficient matrix.

3. Calculation on CPD

The Compact PWI experimental device is a new experimental spherical tokamak device whose main parameters are as follows:

Plasma major radius $R = 0.3$ m
Plasma minor radius $a = 0.2$ m
Toroidal field $B_{\mathrm{t}} = 0.3$ T @ $R = 0.25$ m
Operation period τ_{d} =1.00 sec for $B_{\mathrm{t}} = 0.3$ T
Plasma current $I_{\mathrm{p}} = 150$ kA

The calculation configuration for CPD is shown in Fig.2. There are 7 poloidal field coils including CS and 45 flux loops, and the CCS is an ellipse with minor/major axis 0.06m/0.10m located at plasma center. The ellipse is discretized into 6 points for numerical calculation. The mesh size for calculation is 100x200 and mesh precision is 1cm.

In order to check the calculation precision, an equilibrium code [4] based on Green's function is used to make ideal flux signal whose calculation error is about 0.5%. The main steps of comparison are as follows:

(1) Ideal flux surfaces are made by equilibrium code.
(2) These plasma shapes are reproduced by using CCS method.
(3) The original and reproduced shapes are compared.

The shape difference dif is defined as shown in Fig.3.

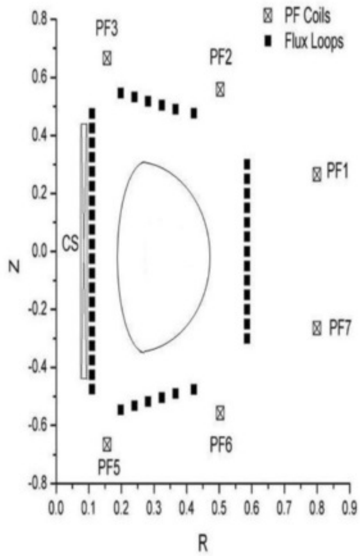
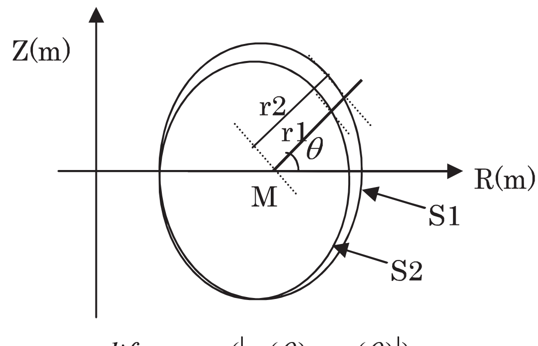


Fig.2 Magnetic sensor positions of CPD

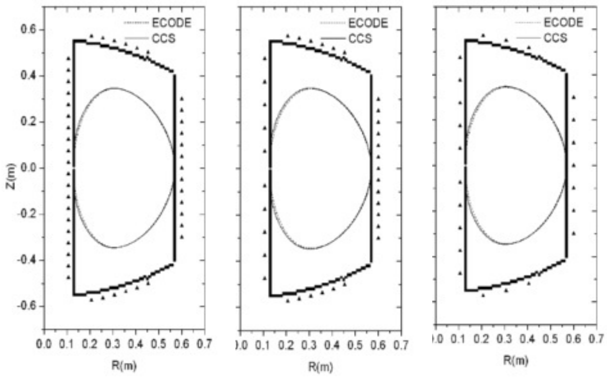


$$dif = \max_{\theta}(|r_2(\theta)-r_1(\theta)|)$$

Fig.3 Plasma shape difference
S1 is original shape, S2 is reproduced shape
M is the center of vacuum chamber

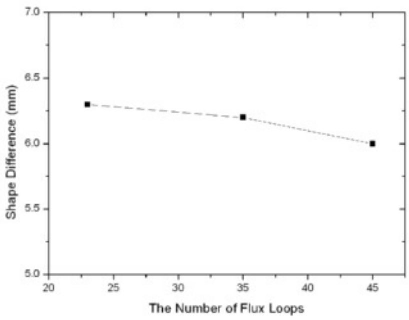
3.1 Flux loop dependence

In the present stage there are 45 flux loops measurements. In order to analyze the flux loop dependence, different numbers of flux loops are used to reproduce the plasma shape. Some new sensor distributions based on the original distribution are added for calculation. Each side of flux loops surface are divided in the same interval, sensors are positioned. The typical results are shown in Fig.4. When the number of flux loops is decreased from 45 to 23, the difference is less than 1cm.



(45 flux loops)　(35 flux loops)　(23 flux loops)

(a) Reconstructed Shape using different numbers of flux loops

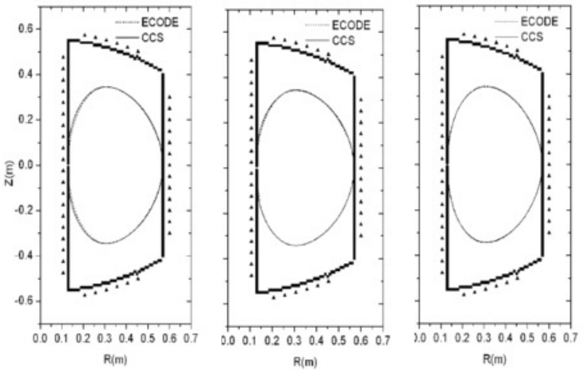


(b) Shape difference of different flux loops

Fig.4 Flux loop dependence

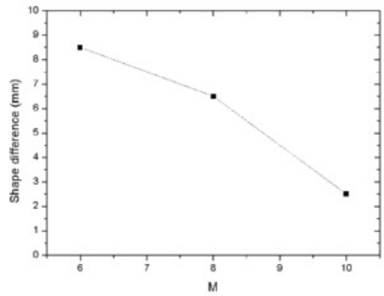
3.2 Cauchy-condition dependence

In case of much elongated and triangular plasmas in spherical tokamak, good precision can be achieved by increase in degrees of parametric freedom representing the Cauchy Condition (M) as shown in Fig. 5. The shape difference decreases from 8.5mm to 2.5mm, when M is increased from 6 to 10.



(M=6)　(M=8)　(M=10)

(a) Reconstructed shape using different M

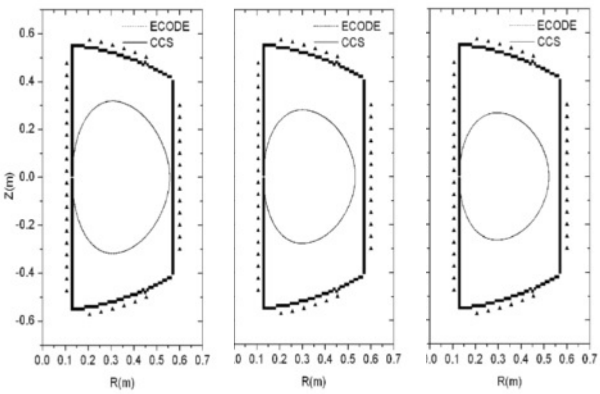


(b) Shape difference of different M

Fig.5 Cauchy-Condition dependence

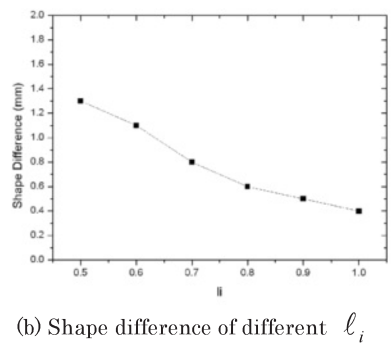
3.3 ℓ_i dependence

Various kinds of plasma current profiles may appear in the real discharge experiments. In order to control plasma shape precisely in real-time, ℓ_i dependence of CCS method to reproduce spherical tokamak plasma shape is studied. Two kinds of plasma shapes are considered. Fig.6 shows a sample comparison of limiter plasma which is not much elongated. The shape difference of plasma shape is less than 1.4mm, when ℓ_i is from 0.5 to 1.0, and the number M is 6.



(ℓ_i =0.8)　(ℓ_i =0.9)　(ℓ_i =1.0)

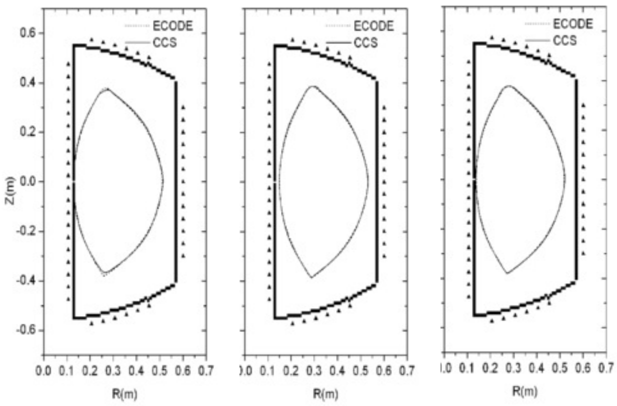
(a) Reconstructed plasma shape of different ℓ_i (β_p = 0.3)



(b) Shape difference of different ℓ_i

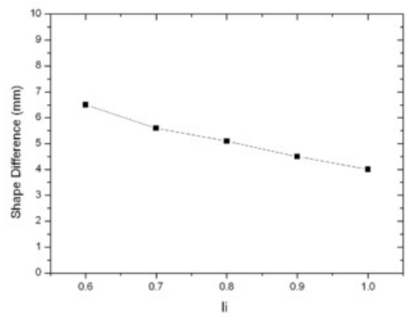
Fig.6 ℓ_i dependence in limiter plasma

In case of double-null plasmas which is much elongated and triangular, typical plasma shapes corresponded to ℓ_i =0.6 ~ 1.0 (βp = 0.5) are calculated. As shown in Fig.7, the shape difference is less than 6.5mm, while M is 6.



(ℓ_i =0.7) (ℓ_i =0.8) (ℓ_i =0.9)

(a) Reconstructed plasma shape of different ℓ_i (β_p= 0.5)



(b) Shape difference of different ℓ_i

Fig.7 ℓ_i dependence of double-null plasma

4. Experimental Data Calculation

4.1 Eddy current analysis

Normally plasma boundary shape control carried out in the flattop stage of plasma discharge shows good results, but at the ramp up and fall down stage, the calculation results are not so good because of large eddy current effect which can not be neglected in the plasma shape reproduction using magnetic measurement. Since it is difficult to know exactly the eddy current profile in analysis region, for example, in vacuum chamber or tokamak structure, that means an exact calculation module can not be made for eddy current. In present stage from the magnetic measurement it is known that the eddy current effect is large in CPD experiment, and there are no special magnetic measurements for eddy current now, so some proper module should be selected to evaluate the eddy current effect.

As shown in Fig.8, new poloidal field coils are assumed to be located at the same position of PF coils, i.e. PF1-new, PF2-new, PF3-new, CS-new, and the effect of these coils consists of two parts, the real PF coils current and the eddy current. The current of these new PF coils are unknown which can be expressed as follow.

$$PF_{inew} = PF_i + I_{eddy}(i) \quad (5)$$

Where,
PF_{inew} is the i_{th} new PF coil current
PF_i is the i_{th} real PF coil current
$I_{eddy}(i)$ is the eddy current function of all eddy current source at i_{th} PF coil

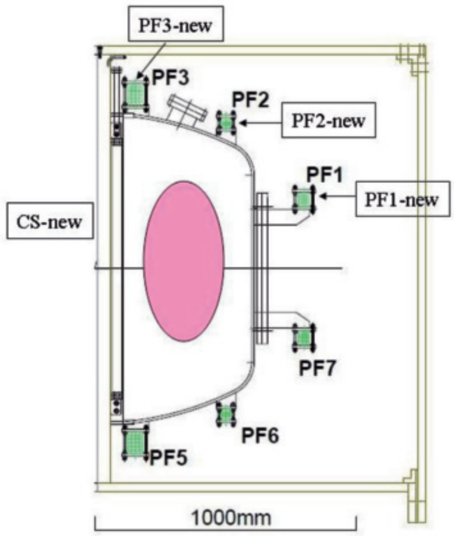


Fig.8 Eddy current effect included in PF coil current

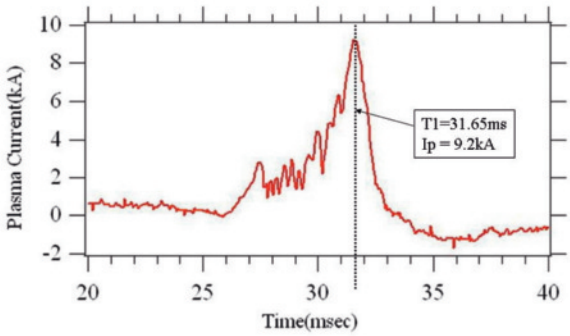
In plasma shape reproduction before, PF coils current is known value from measurement used for input. Now new PF is adopted for calculation instead of PF coils. Then it is not necessary to evaluate eddy current directly, since their effect has been included into these new PF coils. Eddy current effect is not calculated independently, the total effect of eddy current and poloidal field coils current are considered together as some hypothetical poloidal field coils current. The unknown current is solved together with plasma boundary calculation. This is a kind of numerical approximation.

In case of Cauchy-condition surface method, if $N_C + M < N$, where, N_C is the number of PF coils, M is the number of Cauchy point, and

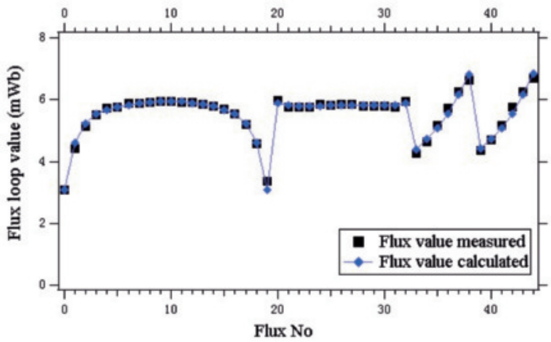
N is the number of magnetic measurement signals, by using least square fitting, the current of new PF coils can be calculated, and at the same time, plasma shape are also reconstructed.

4.2 Sample calculation

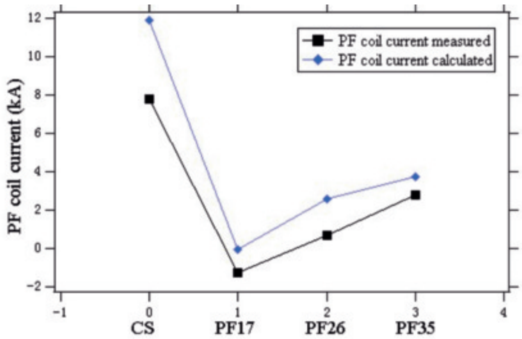
By using the method described above, the experimental shot data are tested. Fig.9 shows a sample plasma shape reconstruction. The shot number is 504101 which is an OH and RF discharge [5].



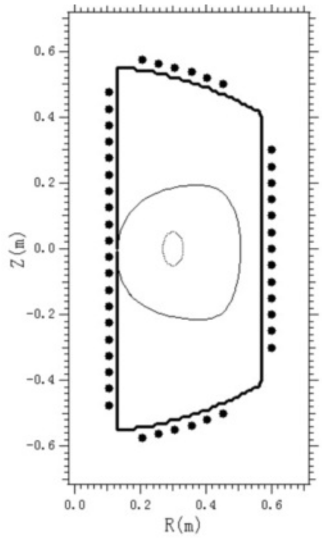
(a) Waveform of plasma current



(b) Flux loop value measured and calculated



(c) PF coil current measured and calculated



(d) Plasma shape reproduced by CCS method

Fig.9 Sample calculation of experimental data

The waveform of plasma current measured by Rogowski coil is shown in Fig.9 (a), and the calculation time for shape reproduction is at the peak time T1=31.65ms, where plasma current is 9.2kA. Fig.9 (b) shows the 45 flux loop value measured and calculated. Fig.9 (c) shows the PF coil current measured and calculated which include eddy current effect. The plasma boundary shape is shown in Fig.9 (d), which is limiter plasma of inverse D shape. The CCS is located at (R=0.30m, Z=0.00m), and minor/major axis is 2a/2b=0.06m/0.10m. The shape is reproduced with all 45 flux loop measurements.

5. Conclusion

The Cauchy Condition Surface method is based on an analytical exact solution for magnetic field in the multiply-connected vacuum region and numerically reconstructs plasma shape with good precision.

The CCS method can reproduce plasma shape of spherical tokamak with different number of flux loops measurement (45, 35 and 23), while the shape difference is less than 1cm. Even in case of much elongated and triangular plasmas in spherical tokamak, good precision can be achieved by increase in degrees of parametric freedom representing the Cauchy condition.

The CCS method can reproduce spherical tokamak plasma shape with good precision in different plasma current profiles (ℓ_i=0.5 ~ 1.0). In case of small elongated plasma, normally the shape difference is less than 3mm, and in case of much elongated and triangular double-null plasma, the shape difference is less than 8mm, while the mesh precision is 1cm.

The real experimental data of plasma discharge is tested by using CCS method. Eddy current effect is considered and plasma shape can be reproduced by CCS method.

For the purpose to carry out real-time plasma shape reconstruction, more precise module of eddy current should be studied.

References

1) K. Kurihara, Nucl. Fusion 33 (1993) 399-412.
2) K. Kurihara, Fusion Engineering and Design 51-52 (2000) 1049-1057.
3) F. Wang, K. Nakamura, O. Mitarai, K. Kurihara, Y. Kawamata, M. Sueoka, Proc. 7th Cross Straits Symposium, Kyushu University (2005) 29-30.
4) T. Takeda and S. Tokuda, Computation of MHD Equilibrium of Tokamak Plasma, J. Computational Phys., Vol. 93, No. 1 (1991) 1-107.
5) S. Watanabe, K. Hanada, Proc. 8th Cross Straits Symposium, Pusan, Korea (2006) 231-232.