Modification of Grey Relational Analysis (GRA) Method for Improved Decision Making

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Abstract: The main purpose of this study is to modify the Grey Relational Analysis (GRA) method by using symmetry points to improve its accuracy in decision-making. This modification aims to enhance the sensitivity of GRA in measuring the proximity between alternatives and the ideal solution by considering symmetry in the data. By applying symmetry points, this research improves the way GRA handles imbalanced data variations, resulting in more accurate and objective analysis. The novelty of this approach lies in the integration of the concept of symmetry points into the calculation of relational coefficients, which has not been previously applied in the context of GRA, thus enabling a more adaptive mapping of the data distribution characteristics. The ranking results from the selection of the best employees show that A4 employees are in first place with a score of 0.1254. The results of the comparison of Spearman's correlation values for five different methods. The highest result was achieved by the GRA-SP method with a correlation value of 1, followed by GRA-Entropy which was close to the maximum value of 0.9667. The GRA-LOPCOW method is in third place with a correlation value of 0.8, while GRA-RS has a correlation of 0.75. The lowest correlation value was found in the GRA-ROC method with 0.575. This shows that the GRA-SP method has the strongest correlation, while the GRA-ROC shows the weakest relationship among the methods compared.

Keywords: Comparison; GRA-SP; Improving; Spearman’s Correlation; Symmetry Point

1. Introduction

Multi-criteria decision-making (MCDM) is an approach used to assist decision-makers in assessing and selecting the best option from several available alternatives, taking into account a variety of competing criteria[1–3]. MCDM is very useful when the decision at hand involves the consideration of various factors that have different weights or interests[4–6]. This method includes various techniques, such as Simple Additive Weighting (SAW), Analytic Hierarchy Process (AHP), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which are used to process data and rank alternatives based on predetermined criteria. With MCDM, decision-makers can achieve more rational and objective solutions in complex situations[7–9]. Efficient and accurate methods of decision-making are essential in various sectors because the right decisions can have a significant impact on an organization's performance and success. In the business sector, good decisions can increase profitability, competitiveness, and customer satisfaction. In the public sector, accurate decision-making can improve the efficiency of public services and resource allocation[10–12]. In the industry, efficient decisions can optimize supply chains, reduce costs, and increase productivity. Efficient methods help decision-makers process information faster, while accuracy ensures that every decision is based on in-depth and valid analysis[13,14]. Therefore, the use of the right methods in decision-making is essential to produce optimal and sustainable results in various sectors[15].

Grey Relational Analysis (GRA) is one of the MCDM methods used to analyze the relationship between various alternatives and criteria, especially when the available data is incomplete or there is uncertainty[16–18]. GRA comes from grey systems theory, which deals with uncertainty in systems with limited data. This method is very effective in dealing with decision-making problems with small sample sizes or imperfect information. GRA has the main advantage of handling data with incomplete or ambiguous information and allows analysis that remains valid even if the data is not perfect, making it flexible in situations with uncertainty or partial data[19–21]. Its advantages include the ability to manage small datasets without losing accuracy, as well as analyzing non-linear relationships between variables. In addition, the process is simple and efficient, making it suitable for use in dynamic and fast-changing systems[16,22]. With this approach, GRA provides optimal results in decision-making even when the information available is limited or ambiguous[23,24].

The GRA method has several limitations, especially in terms of sensitivity to data changes and difficulty in handling high-dimensional data. First, GRA is less sensitive to small changes in data, which means that minor data variations are not always proportionally reflected in the results of the analysis. This can reduce the accuracy of decision-making in situations where small changes to criteria are highly influential. Second, GRA face difficulties when it comes to dealing with high-dimensional data, where many criteria or variables must be considered. Under these conditions, the complexity of the analysis increases, and this method can become less efficient as calculating the grey correlation value for each dimension becomes more difficult. This can slow down the process and degrade the quality of the resulting decisions, especially when multiple criteria have to be accommodated simultaneously. As a solution, it is often necessary to modify or merge the GRA with other methods to address these limitations.

The main objective of this study is to modify the GRA method by using symmetry points to improve its accuracy in decision-making. This modification aims to increase the sensitivity of GRA in measuring the proximity between alternatives and ideal solutions by considering the symmetry in the data[25]. By applying symmetry points, this study is expected to improve the way GRA handles unbalanced data variations, resulting in more precise and objective analysis[26,27]. These modifications are expected to improve accuracy in decision-making, especially on complex and multidimensional data, where balance factors are important in assessing alternatives more holistically. Through the integration of symmetry points in GRA, this method is expected to overcome limitations that are often faced, such as lack of sensitivity to small changes in data and challenges in handling high-dimensional data. Additionally, these modifications allow for improvements in data processing efficiency, making GRA more responsive to data changes and more reliable in generating optimal decisions in a variety of complex decision-making contexts[28–30].

Research from M. Tarek, E. Hamouda, and A. S. Abohamama[31] to combine the GRA method by using the entropy weighting method and determining the weight of indicators more objectively and accurately, as well as to improve the quality of decision-making in situations involving many criteria. The drawbacks of integrating entropy weighting into GRA can add to the complexity of calculations, especially when the number of criteria analysed is very large. This can require more time and resources to calculate the weight of Entropy before applying it in a GRA analysis.

Research from R. Andika[32] applied the decision support system model using a combination of GRA and rank order centroid (ROC) to provide alternative rankings by calculating the rank of each criterion. Although GRA helps handle uncertainty in data, this combination may still be less effective in situations where data is highly ambiguous or unstructured, as GRA and ROC focus more on deterministic analysis. In some cases, the use of ROC to provide ratings can overlook the complex interactions between criteria, which may be important in the context of specific decision-making.

The research conducted by A. A. Izka, and H. Sulistiani[33] on the application of the LOPCOW and GRA logarithmic percentage change-driven objective weighting methods in decision support systems offers a comprehensive approach in prioritization and multi-criteria evaluation. The combination of these two methods provides more accurate and objective solutions in decision-making in various fields. The downside of the combination of the two is that they rely heavily on the available quantitative data. If the data used is incomplete or has inaccuracies, the results of the analysis may be biased or unreliable.

Research conducted by P. Citra, I. Sriyasa and H. B. Santoso[34] The combination of GRA and the Rank Sum weighting method in the decision support system creates a balanced approach between the objectivity of weighting and the analysis of alternative relationships to the ideal solution. The more important criteria get more weight by dividing the number of criteria ranking by the total existing criteria. The downside of this combination of methods is that it depends on the ratings set by the decision-maker, which can be subjective. If the decision-maker is wrong in determining the ranking.

The Entropy and LOPCOW methods are considered superior compared to other objective weighting methods such as WENSLO or CRITIC because of their more in-depth approach and sensitivity to data. The Entropy method calculates weights based on the level of uncertainty or information variation of each criterion, so the greater the variation in the value of a criterion between alternatives, the greater the weight assigned[5]. This makes Entropy very effective in identifying criteria that truly contribute high information in decision-making. On the other hand, the LOPCOW method offers a unique approach by evaluating weights based on stability and the percentage of value changes among alternatives using logarithms. This approach makes LOPCOW more responsive to data dynamics and capable of providing more precise weighting without being significantly affected by outliers[27]. Unlike the CRITIC method, which relies solely on the correlation between criteria and standard deviation, and WENSLO, which tends to be based purely on the dispersion of values, Entropy and LOPCOW provide weighting results that are more representative of the actual characteristics of the data. Therefore, these two methods are often considered more reliable in the context of systems for data-driven decision making.

Based on a literature review in previous studies that modified or combined with the GRA method in terms of criterion weights, there is a weakness in the GRA method, namely the GRA method is not a process in determining the criteria weights. The criterion weights in the GRA method are used based on the subjective approach of decision-makers or use a combination with the criterion weighting method.

Symmetry points refer to the points or positions that represent balance or equality in a data structure, mathematical model, or evaluation system. In multi-criteria decision making, symmetry points can be used to identify the balance between alternatives against ideal and anti-ideal solutions, thus allowing for a more objective and fair analysis[35,36]. By considering symmetry points, evaluation methods can be more sensitive to uneven data distributions and reduce biases caused by the dominance of certain criteria. This approach not only strengthens the rationality aspect in the decision-making process but also enhances the reliability of evaluation outcomes in complex situations with many variables. The advantage of using symmetry points in decision-making analysis is its ability to improve the balance of evaluation among alternatives by considering proportional relationships to ideal and anti-ideal solutions simultaneously. This helps to reduce evaluation bias that often occurs when the data is asymmetric or has uneven distribution. In addition, symmetry points allow the evaluation method to be more sensitive to small changes in the data, thus improving the accuracy and precision in ranking alternatives. This approach also strengthens objectivity in the calculation of distance or closeness to references, as well as supporting more rational and consistent decision-making, especially in cases with many interacting criteria.

Improving the performance of GRA in terms of accuracy, consistency, and its ability to handle high-dimensional data is essential to ensure the effectiveness of this method in decision-making. With modifications that apply symmetry points, GRA is expected to improve the accuracy of the analysis by better capturing variations between alternatives and giving more appropriate weights to different criteria. Consistency of results can also be improved, as this approach helps to reduce the impact of extreme values that can interfere with the overall assessment. In addition, the GRA ability to handle high-dimensional data will increase, as symmetry points allow for more effective processing of information from various criteria. By reducing the complexity of calculations and strengthening the analysis of relationships between alternatives, this modified method will provide more stable and reliable results. Overall, these improvements will allow the GRA to become a more powerful tool in complex and varied decision-making situations, expanding the application of this method across different sectors and research fields.

2. Material and Method

The research flowchart is a visual representation of the stages carried out in a research process, aimed at illustrating the logical and systematic flow from the beginning to the end of the research. By using a flowchart, it becomes easier to understand the structure and sequence of the research steps as a whole, thereby enhancing transparency, efficiency, and replication in the scientific process. The research flowchart that was carried out is shown in Figure 1.

The research process begins with the data collection stage, where relevant information is gathered as the basis for analysis, both in the form of primary and secondary data. Next, identification of alternatives and criteria is conducted, which means determining the options to be evaluated and establishing the assessment criteria used in decision-making. After that, the GRA-SP method is applied to analyze the relationship between alternatives and criteria by considering the points of symmetry in the calculation of gray relational coefficients, thus producing a more objective and balanced assessment. Finally, the sorting or ranking of alternatives is carried out based on the calculated relational values to determine the best alternative according to the research objectives.

The data sources in this research were obtained through primary methods such as observation, interviews, and questionnaires. In addition, it is also important to explain the data preprocessing stages systematically, which includes the data cleaning process to eliminate duplicates or inconsistent values, handling missing data, and normalization to ensure the data is on a comparable scale. This preprocessing process aims to ensure that the data used is valid and ready to be analyzed using the GRA-SP method, so that the final results of the research can be trusted and have a high level of objectivity.

The method used in this study is a combination of GRA

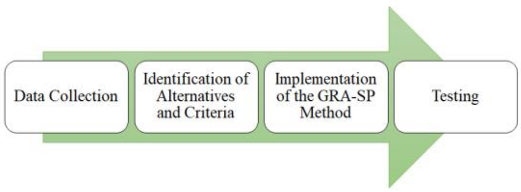

Fig. 1: Research Flow Diagram

and weighting methods to evaluate alternatives based on predetermined criteria. The symmetry point approach is used in the weighting of criteria for alternative evaluation in the GRA method called Grey Relational Analysis-Symmetry Point (GRA-SP). The new GRA method, GRA-SP, provides a comprehensive approach to multi-criteria decision-making, ensuring that the weight of the criteria is determined by considering the equivalence between the criteria, while optimally measuring the performance of alternatives through the GRA. Figure 2 is the stages in the GRA-SP method in problem solving for decision making.

The stages of the GRA-SP method in Figure 1 have 9 stages that are carried out, of which 4 stages are from the GRA method and 5 stages from the results of the modifications made. The GRA-SP method involving the symmetry point stage combines the principle of equilibrium in the weighting of criteria with the analysis of the relative proximity of the alternatives.

The first stage is to create a decision matrix: In this stage, the initial data is collected in the form of a decision matrix that contains the value of each alternative against various criteria. Each row represents an alternative, while each column represents a criterion. This matrix is the basis for further evaluation. The decision matrix is created using the following equation.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{bmatrix}; i = 1,2,..,m; j = 1,2,..,n \qquad (1)$$

The second stage of calculating symmetry points: symmetry points are reference points that are calculated for each criterion to determine the balance in the distribution of data. This point of symmetry can be obtained by calculating the mean or midpoint of the range of values on each criterion. Symmetry points will be used as a benchmark to calculate the absolute distance of each alternative to the point of symmetry calculated using the following equation.

$$SPC_j = \frac{min\{x_{ij}\}+max\{x_{ij}\}}{2} \qquad (2)$$

once the symmetry points are determined, the next step is

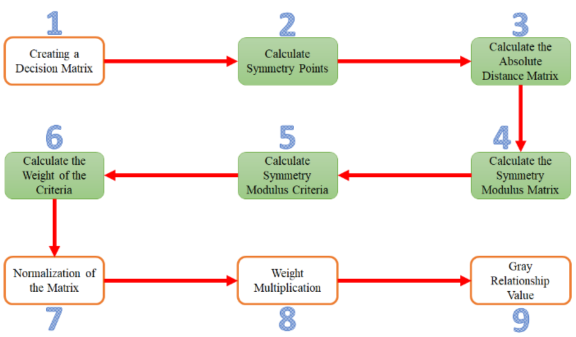

Fig. 2: Framework GRA-SP

to calculate the absolute distance matrix. This matrix contains the absolute difference between the value of each alternative on a criterion and the symmetry point of that criterion. This is done to assess how far the alternative deviates from the ideal equilibrium calculated using the following equation.

$$d_{ij} = \begin{vmatrix} |x_{11} - SPC_1| & \cdots & |x_{1m} - SPC_m| \\ \vdots & \ddots & \vdots \\ |x_{n1} - SPC_1| & \cdots & |x_{nm} - SPC_m| \end{vmatrix} \qquad (3)$$

The fourth stage of the symmetry modulus matrix calculation: the symmetry modulus matrix is calculated by dividing each element in the absolute distance matrix by the maximum value of each criterion. It aims to measure the degree of deviation relative to each alternative to the point of symmetry in a standardized scale calculated using the following equation.

$$r_{ij} = \begin{vmatrix} \frac{\frac{\sum_{i=1}^{m} d_{i1}}{m}}{x_{11}} & \cdots & \frac{\frac{\sum_{i=1}^{m} d_{1m}}{m}}{x_{1j}} \\ \vdots & \ddots & \vdots \\ \frac{\frac{\sum_{i=1}^{m} d_{i1}}{m}}{x_{i1}} & \cdots & \frac{\frac{\sum_{i=1}^{m} d_{nm}}{m}}{x_{ij}} \end{vmatrix} \qquad (4)$$

The fifth stage of the criterion calculation of the symmetry modulus: In this stage, the symmetry modulus value for each criterion is calculated by summing all the values in the column in the symmetry modulus matrix. This value illustrates the magnitude of the relative deviation of all alternatives to the point of symmetry for each criterion calculated using the following equation.

$$q_i = \begin{vmatrix} \frac{\sum_{i=1}^{m} r_{11}}{m} & \cdots & \frac{\sum_{i=1}^{m} r_{1m}}{m} \\ \vdots & \ddots & \vdots \\ \frac{\sum_{i=1}^{m} r_{1n}}{m} & \cdots & \frac{\sum_{i=1}^{m} r_{nm}}{m} \end{vmatrix} \qquad (5)$$

The sixth stage calculates the criterion weight: The criterion weight is calculated based on the value of the criterion's symmetry modulus. Criteria with a smaller symmetry modulus value (less deviation from the point of symmetry) will be given a higher weight, while criteria with a larger deviation will be given a lower weight. This reflects the importance of more balanced criteria in the evaluation calculated using the following equation.

$$w_i = \frac{q_i}{\sum_{i=1}^{m} q_i} \qquad (6)$$

The seventh stage of the matrix normalization calculation: After the criteria weights are calculated, the next step is to normalize the decision matrix. Normalization is carried out so that the values in the matrix are in a uniform range. This normalization allows for a fair comparison between criteria with different scales to be calculated using the following equation.

$$X_{ij} = \frac{x_{ij} - x_{min\,ij}}{x_{max\,ij} - x_{min\,ij}} \quad (7)$$

The eighth stage calculates the multiplication of the weights: The value in the normalization matrix is then multiplied by the weight of the previously calculated criteria. This step integrates the weight of the criteria into the calculation, so that each criterion exerts a weighted influence on the final assessment of the alternative is calculated using the following equation.

$$V_{ij} = x_{i,j} * w_j \quad (8)$$

The last step is to calculate the Grey Relational Grade (GRG) for each alternative. This value shows the relative proximity between each alternative and the ideal solution. The alternative with the highest GRG value is considered the closest to the ideal solution and, as such, preferred in the final decision is calculated using the following equation.

$$GRG_i = \frac{1}{n} \sum_{j=1}^{n} V_{ij} \quad (9)$$

This stage of GRA-SP strengthens the GRA approach by taking into account the balance between criteria through symmetry points, as well as providing more objective weighting based on the relative deviation of the alternative to the symmetry point.

3. Result and discussion

The GRA-SP method introduces a number of modifications to the traditional GRA method to improve its performance in decision-making scenarios, especially in handling complex, high-dimensional, or uncertain data. The core modifications of this approach focus on increasing the weighting of the criteria as well as how to calculate the relative proximity of alternatives to the ideal solution. In this modification, the weight of the criteria is determined based on the balance or symmetry between all the criteria used in the decision-making process. The Symmetry Point method ensures that no criterion has too much weight over the other, so each criterion is considered to have a balanced level of importance.

GRA-SP offers a solution to the problem of subjectivity in weighting by creating points of symmetry between criteria. In this approach, the weight of the criteria is arranged in such a way that the distribution of the weights is fairer and no criteria are ignored or over-prioritized. This modification is particularly suitable for decision-making situations where all criteria are considered important, and a balance in the weight of the criteria can result in a more comprehensive decision. These modifications also increase the transparency and interpretability of decisions as each criterion contributes proportionally to the final outcome, allowing decision-makers to avoid any bias that may arise from disproportionately assigning subjective weight. With a symmetry point approach, GRA can be used in a variety of decision-making contexts that require fairness in the weight of criteria, such as performance evaluation, product selection, and risk assessment. The GRA-SP modification brings innovation in improving consistency and fairness in multi-criteria analysis, which makes it more reliable and flexible to apply to decision-making scenarios that require balanced criteria weighting.

3.1. Case study

In this study, the case study used in the process of selecting the best employees is one of the important efforts to increase motivation and reward individuals who contribute greatly to organizational achievements. The criteria used in selecting the best employees are K1 is performance and productivity, K2 is commitment and work ethic, K3 is job skills, K4 is innovation and creativity, K5 is teamwork, K6 is leadership skills.

Alternative assessment data for the selection of the best employees is information that contains the evaluation of various employees based on a number of criteria used to assess their performance objectively. These criteria usually include aspects such as predetermined performance. Each employee is assigned a score for each criterion, which can then be compared to determine which employees have the best overall performance. The data of alternative assessment of employee selection are shown in Table 1.

The results of the collection of alternative assessment data for the selection of the best employee are a summary of the assessment that shows the performance of each employee based on the criteria that have been set. Once the data from each employee is collected and assessed based on each criterion, these results provide a clear picture of how each employee compares to each other. These results provide objective and measurable policies in the decision-making process. The best employee selection data in Table 1 is the primary data used in this study, obtained from CV companies. XYZ which is in Bandar Lampung. This data will be analysing using the GRA-SP method and the results will be compared with the rankings obtained from data collection.

Table 1: Alternative assessment data

Alternative Name	K1	K2	K3	K4	K5	K6
A1	9	8	9	7	8	9
A2	8	7	8	9	9	8
A3	7	9	7	8	9	7
A4	9	9	9	8	7	9
A5	8	7	8	8	8	7
A6	7	8	7	9	9	8
A7	9	9	8	8	7	7
A8	8	8	9	7	8	8
A9	9	8	7	9	9	9

3.2. Implementation of GRA-SP Method

The implementation of the GRA-SP method is an innovative step in increasing the effectiveness of decision-making in various fields that involve alternative evaluations based on complex criteria. This method combines the basic principles of grey relational analysis with the point of symmetry approach, which allows for a more objective and balanced determination of criterion weights. By calculating the absolute distance between the alternative and the ideal point of symmetry, as well as performing standardized normalization and evaluation, the GRA-SP provides a more accurate picture of the proximity of the alternative to the optimal solution.

The first step is to create a decision matrix based on the assessment data in Table 2 using equation (1).

$$X = \begin{bmatrix} 9 & 8 & 9 & 7 & 8 & 9 \\ 8 & 7 & 8 & 9 & 9 & 8 \\ 7 & 9 & 7 & 8 & 9 & 7 \\ 9 & 9 & 9 & 8 & 7 & 9 \\ 8 & 7 & 8 & 8 & 8 & 7 \\ 7 & 8 & 7 & 9 & 9 & 8 \\ 9 & 9 & 8 & 8 & 7 & 7 \\ 8 & 8 & 9 & 7 & 8 & 8 \\ 9 & 8 & 7 & 9 & 9 & 9 \end{bmatrix}$$

The second stage calculates symmetry points as a benchmark to calculate the absolute distance of each alternative to the symmetry point using equation (2).

$$SPC_1 = \frac{min\{x_{11}, x_{19}\} + max\{x_{11}, x_{19}\}}{2} = \frac{7+9}{2} = 8$$

$$SPC_2 = \frac{min\{x_{21}, x_{29}\} + max\{x_{21}, x_{29}\}}{2} = \frac{7+9}{2} = 8$$

$$SPC_3 = \frac{min\{x_{31}, x_{39}\} + max\{x_{31}, x_{39}\}}{2} = \frac{7+9}{2} = 8$$

$$SPC_4 = \frac{min\{x_{41}, x_{49}\} + max\{x_{41}, x_{49}\}}{2} = \frac{7+9}{2} = 8$$

$$SPC_5 = \frac{min\{x_{51}, x_{59}\} + max\{x_{51}, x_{59}\}}{2} = \frac{7+9}{2} = 8$$

$$SPC_6 = \frac{min\{x_{61}, x_{69}\} + max\{x_{61}, x_{69}\}}{2} = \frac{7+9}{2} = 8$$

The third stage calculates the absolute distance matrix to assess how far the alternative deviates from the ideal equilibrium by using equation (3).

$$d_{11} = |x_{11} - SPC_1| = |9 - 8| = 1$$

The overall result of the absolute distance matrix value is shown in Table 2.

The fourth stage calculates the symmetry modulus matrix using equation (4).

$$r_{11} = \left| \frac{\frac{\sum_{i=1}^{m} d_{11,19}}{9}}{x_{11}} \right| = \left| \frac{\frac{6}{9}}{9} \right| = 0.074$$

Table 2: The overall result of the absolute distance matrix value

Alternative Name	K1	K2	K3	K4	K5	K6
A1	1	0	1	1	0	1
A2	0	1	0	1	1	0
A3	1	1	1	0	1	1
A4	1	1	1	0	1	1
A5	0	1	0	0	0	1
A6	1	0	1	1	1	0
A7	1	1	0	0	1	1
A8	0	0	1	1	0	0
A9	1	0	1	1	1	1

Table 3: The overall result of the symmetry modulus matrix value

Alternative Name	K1	K2	K3	K4	K5	K6
A1	0.074	0.069	0.074	0.079	0.083	0.074
A2	0.083	0.079	0.083	0.062	0.074	0.083
A3	0.095	0.062	0.095	0.069	0.074	0.095
A4	0.074	0.062	0.074	0.069	0.095	0.074
A5	0.083	0.079	0.083	0.069	0.083	0.095
A6	0.095	0.069	0.095	0.062	0.074	0.083
A7	0.074	0.062	0.083	0.069	0.095	0.095
A8	0.083	0.069	0.074	0.079	0.083	0.083
A9	0.074	0.069	0.095	0.062	0.074	0.074

The overall result of the symmetry modulus matrix value is shown in Table 3.

The fifth stage calculates the symmetry modulus of the criterion using equation (5).

$$q_1 = \left| \frac{\sum_{i=1}^{m} r_{11}, r_{19}}{m} \right| = \left| \frac{0.737}{9} \right| = 0.0819$$

$$q_2 = \left| \frac{\sum_{i=1}^{m} r_{21}, r_{29}}{m} \right| = \left| \frac{0.622}{9} \right| = 0.0691$$

$$q_3 = \left| \frac{\sum_{i=1}^{m} r_{31}, r_{39}}{m} \right| = \left| \frac{0.758}{9} \right| = 0.0842$$

$$q_4 = \left| \frac{\sum_{i=1}^{m} r_{41}, r_{49}}{m} \right| = \left| \frac{0.622}{9} \right| = 0.0691$$

$$q_5 = \left| \frac{\sum_{i=1}^{m} r_{51}, r_{59}}{m} \right| = \left| \frac{0.737}{9} \right| = 0.0819$$

$$q_6 = \left| \frac{\sum_{i=1}^{m} r_{61}, r_{69}}{m} \right| = \left| \frac{0.758}{9} \right| = 0.0842$$

The sixth stage of calculating the criterion weight is calculated based on the value of the symmetry modulus of the criterion using equation (6).

$$w_1 = \frac{q_1}{\sum_{i=1}^{m} q_{1,6}} = \frac{0.0819}{0.4703} = 0.1741$$

$$w_2 = \frac{q_2}{\sum_{i=1}^{m} q_1, q_6} = \frac{0.0691}{0.4703} = 0.1469$$

$$w_3 = \frac{q_3}{\sum_{i=1}^{m} q_1, q_6} = \frac{0.0842}{0.4703} = 0.1791$$

$$w_4 = \frac{q_4}{\sum_{i=1}^{m} q_1, q_6} = \frac{0.0819}{0.4703} = 0.1469$$

$$w_5 = \frac{q_5}{\sum_{i=1}^{m} q_1, q_6} = \frac{0.0819}{0.4703} = 0.1741$$

$$w_6 = \frac{q_6}{\sum_{i=1}^{m} q_1, q_6} = \frac{0.0842}{0.4703} = 0.1791$$

The seventh stage calculates the normalization value of the matrix for a fair comparison between criteria with different scales using equation (7).

$$X_{11} = \frac{x_{11} - x_{min\,11,min19}}{x_{max\,11,max\,19} - x_{min\,11,min19}} = \frac{9-7}{9-7} = 1$$

The overall results of the calculation of the matrix normalization value are shown in Table 4.
The eighth stage calculates the value of the multiplication of the weight between the criterion weight and the normalization of the matrix using equation (8).

$$V_{11} = x_{11} * w_1 = 1 * 0.1741 = 0.1741$$

The overall result of the calculation of the weight multiplication result is shown in Table 5.
The first step is to create a decision matrix based on the assessment data in Table 2 using equation (1).

$$GRG_1 = \frac{1}{6} \sum_{j=1}^{n} V_{11,61} = \frac{1}{6} * 0.6927 = 0.1154$$

The overall results of the calculation of the grey relationship value are shown in Table 6. In the process of selecting the best employees, the ranking results using the GRA-SP method provide a clear picture of the position of each prospective employee based on the evaluation criteria that have been determined. Figure 3 is the ranking result of the selection of the best employees.

Table 4: The overall result of the calculation of the matrix normalization value

Alternative Name	K1	K2	K3	K4	K5	K6
A1	1.0	0.5	1.0	0.0	0.5	1.0
A2	0.5	0.0	0.5	1.0	1.0	0.5
A3	0.0	1.0	0.0	0.5	1.0	0.0
A4	1.0	1.0	1.0	0.5	0.0	1.0
A5	0.5	0.0	0.5	0.5	0.5	0.0
A6	0.0	0.5	0.0	1.0	1.0	0.5
A7	1.0	1.0	0.5	0.5	0.0	0.0
A8	0.5	0.5	1.0	0.0	0.5	0.5
A9	1.0	0.5	0.0	1.0	1.0	1.0

Table 5: The overall result of the calculation of the weight multiplication result

Alternative Name	K1	K2	K3	K4	K5	K6
A1	0.1741	0.0734	0.1791	0.0000	0.0870	0.1791
A2	0.0870	0.0000	0.0895	0.1469	0.1741	0.0895
A3	0.0000	0.1469	0.0000	0.0734	0.1741	0.0000
A4	0.1741	0.1469	0.1791	0.0734	0.0000	0.1791
A5	0.0870	0.0000	0.0895	0.0734	0.0870	0.0000
A6	0.0000	0.0734	0.0000	0.1469	0.1741	0.0895
A7	0.1741	0.1469	0.0895	0.0734	0.0000	0.0000
A8	0.0870	0.0734	0.1791	0.0000	0.0870	0.0895
A9	0.1741	0.0734	0.0000	0.1469	0.1741	0.1791

Table 6: The overall result of the calculation of the grey relation for each alternative

Alternative Name	Grey Value
A1	0.1154
A2	0.0978
A3	0.0657
A4	0.1254
A5	0.0562
A6	0.0807
A7	0.0807
A8	0.0860
A9	0.1246

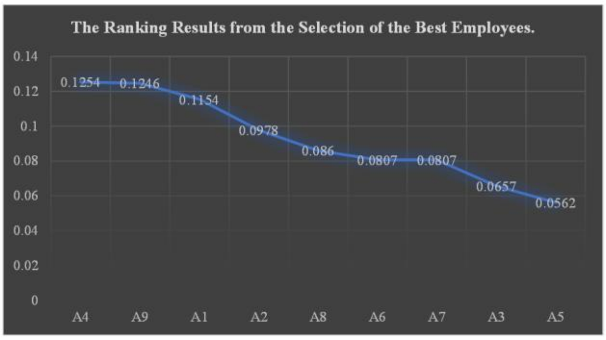

Fig. 3: The ranking results from the selection of the best employees

The ranking results from the selection of the best employees show that A4 Employees are in first place with a value of 0.1254, followed by A9 with a slightly lower value of 0.1246. Employee A1 ranks third with a score of 0.1154. The decline in value continued in A2 employees who obtained a score of 0.0978, followed by A8 with a score of 0.086. employees A6 and A7 are in the same position with a value of 0.0807, indicating that both have a balanced performance. The ranking continued to decline with A3 employees having a score of 0.0657, and A5 employees being in last position with a score of 0.0562. Overall, the results of this ranking indicate that there is variation in performance among employees, with some employees significantly superior to others.

3.3. Sensitivity Analysis of Criteria Weights

Sensitivity analysis of criterion weights is an important stage in the multi-criteria decision-making process, aimed at evaluating to what extent changes in criterion weights affect the final results of alternative ranking. In the context of decision support systems, criterion weights play a crucial role as they reflect the relative importance of each assessed aspect. Through sensitivity analysis, researchers or decision-makers can identify which criteria most influence the decision outcomes and assess the consistency of the method used in ranking. This process also helps test the robustness of the model and detect potential biases that may arise due to the dominance of one or several criteria. Thus, sensitivity analysis serves not only as a validation tool but also provides deep insights for improving decision-making models to be more objective, adaptive, and in accordance with situational needs.

The sensitivity analysis of the criteria weights in this study uses scenarios of gradually increasing and decreasing weights to observe their impact on the final ranking of alternatives. One of the commonly applied scenarios is to increase or decrease the weight of one criterion by 0.05 while keeping the total weight valued at 1. This aims to simulate changes in perception or priorities of decision-makers towards a criterion without disturbing the overall contribution balance. This approach allows for observing the system's sensitivity to realistic small changes in the context of real decision-making. By applying this scenario, any change in weight on one criterion is balanced by a proportional adjustment in the weights of other criteria, ensuring the system remains within a framework of fair and consistent evaluation. The analysis results from this scenario provide insight into the stability of alternative rankings as well as the identification of criteria that most influence the final decision. If a small change such as ±0.05 in one criterion results in a significant change in ranking, then that criterion is considered highly sensitive and needs to be considered more carefully. Thus, this scenario not only tests the resilience of the decision model but also helps refine the allocation of criterion weights more strategically. The ranking results based on 12 weight change scenarios are shown in Figure 4.

The results of the sensitivity analysis on the alternative

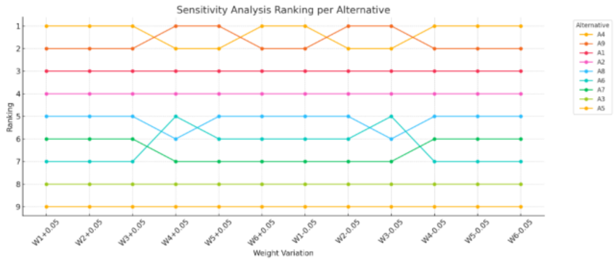

Fig. 4: Alternative ranking results from changes in criteria weights

rankings based on variations in the criteria weights of ±0.05, while keeping the total overall weight equal to 1. This graph illustrates how the ranking positions of each alternative (A1 to A9) change or remain stable when the weights of the respective criteria (W1 to W6) are increased and decreased by 0.05. The horizontal axis represents the variations in weight (increments and decrements), while the vertical axis shows the ranks from 1 (best) to 9 (lowest). The results of the graph show that several alternatives such as A4, A9, and A1 have a very stable ranking at the top positions (ranked 1 to 3) despite changes in weights, which indicates that these alternatives have consistent and robust performance against weight variations. In contrast, alternatives such as A6 and A7 show significant ranking fluctuations, especially at variations W3±0.05 and W4±0.05, indicating that the performance of these alternatives is very sensitive to changes in weights on those criteria. This pattern provides important information for decision-makers in evaluating the stability of selected alternatives against changes in preferences or weight policies.

3.4. Discussion

The GRA method is one of the popular methods in multi-criteria decision making, especially due to its ability to handle uncertain, incomplete, and small-scale data. Although effective in many cases, this method has limitations that create a need for further expansion, especially in facing the challenges of complexity and modern data dynamics. First, standard GRA uses a linear and one-dimensional approach in calculating the closeness between alternatives, without considering symmetry, imbalance, or extreme value distributions. This has the potential to produce rankings that are biased or do not reflect the true reality in situations with large data variations or inter-criteria dependency patterns. Second, traditional GRA methods often rely on subjective weighting or pre-determined weights, which can reduce the objectivity of the results. In modern decision-making, an adaptive and data-driven weighting approach is needed, such as integration with objective methods (Entropy, CRITIC, LOPCOW) to enhance the accuracy of criterion weights. Thirdly, GRA is not yet optimal to be applied in the context of big data, real-time decision-making, or in highly dynamic environments. Without expansion, GRA will lag behind and be less responsive to rapid changes and complex analytical needs.

The expansion of GRA with the Symmetry Point approach is an important step in improving the accuracy, fairness, and sensitivity of analyses in multi-criteria decision making. In traditional GRA methods, the assessment of alternatives is only based on their proximity to the ideal solution (grey relational grade towards the best value), without considering the balance of positions against the worst value or the midpoint. This can lead to imbalances in evaluation, especially when extreme values significantly affect the distances between alternatives. By integrating the Symmetry Point, which is the conceptual midpoint between ideal and non-ideal values, the evaluation becomes more balanced and representative. Each alternative is evaluated not only based on its proximity to the best solution but also on how symmetrical its position is relative to the entire range of values. This approach helps reduce distortion caused by outliers and increases sensitivity to small differences among alternatives, especially in cases where the data among alternatives is within a narrow yet competitive range. Moreover, the extension of GRA using the Symmetry Point also allows for more adaptive integration with objective weighting techniques and enhances the method's resilience to uneven data distribution. This approach enriches the GRA's capability to handle more complex decision-making contexts, such as evaluation based on uncertain data, real-time decision-making, or intelligent systems based on big data. Therefore, the expansion of GRA with the Symmetry Point concept is a strategic step to strengthen the methodological foundation of GRA to remain relevant and excel in facing modern decision-making challenges.

The GRA method has proven to be an effective technique in multi-criteria decision-making, but there are some limitations that need to be overcome, especially when faced with high and complex data. The modifications proposed in this study aim to address these weaknesses and increase the effectiveness of GRA in supporting decision-making. The proposed modifications to the GRA-SP method show great potential in improving the quality of decision-making. Through the integration of better normalization techniques, objective weighting, and efficient algorithms, New GRA is expected to answer existing challenges and become a more reliable tool in the multi-criteria decision-making process in various domains.

In the context of multi-criteria decision-making, previous research combined the GRA method with the Entropy[31)], GRA-Entropy is a combination of the GRA method with weighting using Entropy. Entropy is used to objectively determine the weight of each criterion based on the degree of irregularity or variation of the data. This method aims to reduce subjectivity in determining weights, so that the results of GRA analysis become more objective. By using Entropy, GRA-Entropy can provide a fairer assessment when there are significant differences in the data between the criteria. The advantages of GRA-SP modification in decision-making are as follows:

Balance in Criterion Weighting: The Symmetry Point approach ensures that each criterion has a balanced weight, so that no one criterion dominates the final result too much. This is important when all criteria are considered to be of relatively equal importance.

Reduces Subjectivity: By using a symmetry approach, this modification reduces the influence of subjectivity that often occurs in the determination of criterion weighting. This makes the decision-making process more objective and fair.

Flexibility for Multiple Scenarios: GRA-SP is suitable for use in a variety of decision-making contexts where fairness in the weight of criteria is critical, such as performance evaluation, selection of the best alternative, or risk analysis. This method is flexible and can be applied to a variety of industries and scenarios.

Transparency and Easy to Understand: This method results in more transparent decisions because each criterion has a proportional contribution. This makes it easy for users to understand how each criterion affects the outcome of a decision.

Avoiding Bias: With balanced weight, GRA-SP can reduce the risk of bias that may arise from subjective judgments or personal preferences, resulting in more neutral decisions.

Suitable for Criteria of Equal Importance: This method is ideal for use when no criteria is considered more important than the other. It provides a fairer solution when all criteria have an equal contribution to the outcome of the decision.

Reduced Complexity in Weighting: GRA-SP simplifies the weighting process, as it does not require complex calculations or heavy preference assessments. This makes this method easier to apply in quick decision-making situations.

Overall, the GRA-SP modification offers fairness, objectivity, and ease of use in decision-making, especially when the criteria have a balanced level of importance.

In the context of multi-criteria decision-making, previous research combined the GRA method with the ROC[32)], GRA-ROC is a combination of GRA with the ROC weighting method. ROC is a weighting technique that calculates weight based on a predetermined criterion ranking. More important criteria will be given higher weight. When combined with the GRA, the ROC allows for more intuitive weight-based decision-making, based on the order of priority given to each criterion, but still within the framework of the GRA grey relationship analysis.

In the context of multi-criteria decision-making, previous research combined the GRA method with the LOPCOW[33)], GRA-LOPCOW is an approach that combines GRA with the LOPCOW weighting method. LOPCOW is a weighting technique that combines subjective weight (from expert opinion) and objective weight (from data). This combination strengthens the GRA with a balanced weight of criteria between subjective and objective views, so that the resulting decisions can be more comprehensive and reflect various viewpoints in the evaluation of the criteria.

In the context of multi-criteria decision-making, previous research combined the GRA method with the Rank Sum weighting methods[34)], GRA-Rank Sum uses the Rank Sum weighting approach in GRA. Rank Sum is a simple method that determines the weight based on the criterion ranking,

Table 7: The result of a comparison of the ranking of each alternative

Alternative Name	Original	GRA-Entropy	GRA-ROC	GRA-LOPCOW	GRA-Rank Sum	GRA-SP
A1	1	1	1	2	1	1
A2	2	2	5	1	4	2
A3	3	3	3	5	2	3
A4	4	4	6	4	6	4
A5	5	6	5	7	5	5
A6	6	5	9	3	8	6
A7	7	8	2	8	3	7
A8	8	7	8	6	7	8
A9	9	9	7	9	9	9

with the weight calculated from the number of criterion ratings. This combination gives weight that is easy to calculate and puts more emphasis on criteria that have higher rankings. GRA-Rank Sum is suitable for use when decision-makers want weights that are less complicated and are based on qualitative judgment.

In this study, using GRA-SP refers to the use of the symmetry point method in weighting criteria on GRA. Symmetry Point is a technique that balances weights between criteria by finding a point of symmetry where all criteria are considered to have a balanced level of importance. The GRA-SP ensures that the gray relationship analysis treats all criteria equally, which is useful in cases where no criterion is considered more important than the other. Table 7 is the result of a comparison of the ranking of each alternative from each existing method.

The results of the ranking comparison based on various methods show that A4 ranks first in almost all methods, except for GRA-LOPCOW where A4 ranks second. This shows that A4 is consistently the best candidate based on multi-criteria assessment. A9 also performed well with a second rank on the majority of methods, except for the GRA-Entropy and GRA-ROC methods where the rankings were lower, i.e. fifth and fourth. A1 ranks third on most methods, but its performance declines in GRA-LOPCOW (fifth place).

Spearman correlation is a statistical method used to measure the strength and direction of the relationship between two variables based on their order (ranking). Unlike Pearson's correlation, which measures linear relationships, Spearman correlation calculates how well the relationship between two data sets can be expressed by monotonous functions. This method is often used when the data does not meet the assumptions of normality or linear relationships, making it more flexible for ordinal data or non-linear data. The results of the ranking comparison using spearman correlation are shown in Figure 5.

The results of the comparison of Spearman's correlation values for five different methods. The highest result was achieved by the GRA-SP method with a correlation value of 1, followed by GRA-Entropy which was close to the maximum value of 0.9667. The GRA-LOPCOW method is in third place with a correlation value of 0.8, while GRA-RS has a correlation of 0.75. The lowest correlation value was found in the GRA-ROC method with 0.575. This shows that the GRA-SP method has the strongest correlation, while the GRA-ROC shows the weakest relationship among the methods compared.

The implications of the findings of this research for future research and practice are very significant. Academically, the results obtained open up opportunities for the development of more adaptive and accurate advanced methods in multi-criteria decision making, particularly in the context of the application of the GRA-SP method. These findings can serve as a foundation for further studies that seek to test the effectiveness of similar methods in different domains or sectors. Meanwhile, in practice, this approach can assist decision-makers in various fields, such as management, engineering, and public policy, in obtaining evaluation results that are more objective, balanced, and reliable. Thus, this research not only contributes theoretically but also provides practical benefits that can be directly applied in real-world contexts.

Although the proposed GRA-SP model offers improvements in terms of assessment balance and objectivity, there are several limitations that need to be noted. One of the main limitations is the model's sensitivity to the normalization scale and the selection of the symmetry point, which can affect the stability of the results if not determined precisely. Additionally, this model may be less optimal when applied to data that is highly dynamic

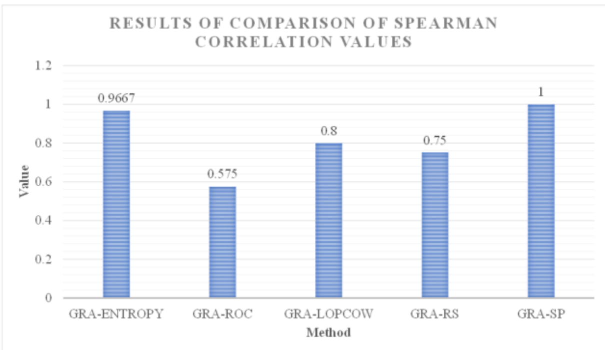

Fig. 5: The results of comparison of spearman correlation values

impact the final ranking accuracy and the effectiveness of the decisions made. These limitations can cause distortions in the calculation of gray relational coefficients, especially if the input data has outliers or an uneven distribution. In addition, the use of fixed symmetry points in all cases can reduce the model's flexibility in adapting to different problem contexts. Therefore, it is important for future research to develop adaptive mechanisms in determining symmetry points and to test the model's resilience to variations in data and complex criteria structures, in order to improve the accuracy and reliability of GRA-SP-based decision support systems.

4. Conclusion

The application of the GRA-SP method is an innovative step in improving the effectiveness of decision-making in various fields that involve alternative evaluations based on complex criteria. This method combines the basic principles of grey relational analysis with the point of symmetry approach, which allows for a more objective and balanced weighting of criteria. The ranking results from the selection of the best employees show that A4 employees are in first place with a score of 0.1254, followed by A9 with a slightly lower score of 0.1246. Employee A1 is in third place with a score of 0.1154. The results of the comparison of Spearman's correlation values for five different methods. The highest result was achieved by the GRA-SP method with a correlation value of 1, followed by GRA-Entropy which was close to the maximum value of 0.9667. The GRA-LOPCOW method is in third place with a correlation value of 0.8, while GRA-RS has a correlation of 0.75. The lowest correlation value was found in the GRA-ROC method with 0.575. This shows that the GRA-SP method has the strongest correlation, while the GRA-ROC shows the weakest relationship among the methods compared. The results of the sensitivity analysis conducted show that several alternatives such as A4, A9, and A1 have stable rankings even with a criterion weight variation of ±0.05. This indicates that the decision model is quite robust, but there are still alternatives that are sensitive to certain weight changes and require more attention in the decision-making process. For future work, this GRA modification method can be tested on various types of data, both from different industries and more complex decision-making situations, such as time-series data or big data. This will test the generalization and flexibility of the GRA-SP method, as well as the potential of this method in rapid decision-making, integrating the New GRA with real-time systems, such as in the context of crisis management or automated control, could be an area of interest. This research shows that the implementation of the GRA-SP method can improve objectivity and balance in the multi-criteria decision-making process. The main findings indicate that the symmetry point-based approach yields more stable results compared to traditional GRA methods. For future research, it is recommended to explore the application of GRA-SP on dynamic or real-time data as well as integration with machine learning-based weighting methods.

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