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A Technique for Calculating Magnetotelluric Impedance Based on the Presence of Noise

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The magnetotelluric method has been widely used for the geophysical investigation and monitoring. Prompted by growing demand, investigators often record magnetotelluric data near urban areas significantly disturbed by cultural noise. To cope with low-quality data, we have developed a calculation technique for deducing the magnetotelluric impedance based on the presence of noise. It can robustly separate the signal and the noise using a hybrid constrained least-squares method. We validate the applicability by examining its performance using real data and several synthetic examples. Our results demonstrate that the technique is effective in working out the magnetotelluric impedance from low-quality spectra.

1. Introduction

The magnetotelluric (MT) method[1)] is an electromagnetic technology to image the resistivity structure of the earth's subsurface according to the relation between natural electric and magnetic spectra in the ground. The depths covered for investigation range from a few hundred meters to several tens of kilometers. In most cases, the MT method is applied for the investigation or monitoring of hydrocarbon resources[2-4)], geothermal reservoirs[5-7)], mineral deposit[8-10)], and disaster-prevention works[11-13)]. Magnetotelluric data are often recorded near urban areas nowadays, mainly to accommodate the growth in demand for disaster-prevention works. The MT method, however, uses weak natural signals, spectrum data converted from MT time-series data. Measurements near cities can thus be flawed, as the data are vulnerable to disturbance by strong cultural noise. Many researchers have devised methods for noise reduction[14-17)] and editing[18)] in an effort to improve the quality of the magnetotelluric spectrum in urban areas. Often, however, we encounter recording points where the quality has yet to improve to the levels we desire. To cope with low-quality spectra, we have developed a technique to calculate the magnetotelluric impedance based on the presence of noise in the spectrum. This technique can robustly separate the signal and noise in the impedance using a hybrid constrained least-squares method. In this study we present calculation results using real data and several synthetic examples to validate the applicability of the technique.

2. Calculating the impedance tensor

2.1 Conventional technique

In the conventional technique for calculating the impedance tensor of remote references[19)], magnetotelluric tensor equations (1) and (2) take the forms of (3), (4), (5), and (6) using the average value of the cross power spectrum.

$$ZxxHx + ZxyHy = Ex \tag{1}$$

$$ZyxHx + ZyyHy = Ey \tag{2}$$

$$Zxx\overline{\left\langle Hx \cdot N^{*}\right\rangle} + Zxy\overline{\left\langle Hy \cdot N^{*}\right\rangle} = \overline{\left\langle Ex \cdot N^{*}\right\rangle} \tag{3}$$

$$Zxx\overline{\left\langle Hx \cdot M^{*}\right\rangle} + Zxy\overline{\left\langle Hy \cdot M^{*}\right\rangle} = \overline{\left\langle Ex \cdot M^{*}\right\rangle} \tag{4}$$

$$Zyx\overline{\left\langle Hx \cdot N^{*}\right\rangle} + Zyy\overline{\left\langle Hy \cdot N^{*}\right\rangle} = \overline{\left\langle Ey \cdot N^{*}\right\rangle} \tag{5}$$

$$Zyx\overline{\left\langle Hx \cdot M^{*}\right\rangle} + Zyy\overline{\left\langle Hy \cdot M^{*}\right\rangle} = \overline{\left\langle Ey \cdot M^{*}\right\rangle} \tag{6}$$

Where E and H are the complex electric and magnetic spectra, respectively, Z is the complex impedance, M^* and N^* are the complex conjugates of the referenced spectrum, and $\overline{\langle\ \rangle}$ is the average value of the cross power spectrum. These equations can uniquely solve the impedance tensor. This calculation technique is effective when only limited noise remains in the remote referenced cross power spectrum. Yet as the noise increases, the quality of the calculated impedance tensor degrades commensurately.

2.2 Technique based on the presence of noise

From here we will present the calculation technique to work out the impedance tensor based on the presence of noise in the referenced cross power spectrum.

First, the constrained least-squares method is applied to consider the presence of noise. Next, the system is solved by the modified Gram-Schmidt method[20)], a method capable of removing a random component as a residual error. We propose the following two calculation techniques based on the constrained least-squares method, defining the noise as a Gaussian random component.

(1) A technique to remove the noise with the residual only.
(2) A technique to remove the noise with both the residual and an unknown noise parameter.

2.2.1 Technique to remove the noise with the residual

This technique relies on tensor equations and smoothness constraints using a Laplacian filter, as shown in (7).

$$\begin{bmatrix} WH \\ \alpha C \end{bmatrix} \cdot \{Z\} = \begin{Bmatrix} WE \\ O \end{Bmatrix} \tag{7}$$

Where H is a cross power matrix of magnetic spectra; E is a cross power vector of electric spectra; Z is an unknown vector of impedance; C is a matrix of a Laplacian filter to constrain impedance to be continuous in the direction of frequency; O is a zero vector; W is a weight matrix based on the spectrum quality, and α is a smoothness factor.

To prevent the sounding curve from becoming a meaningless steep slope or overly smoothed, the cross power spectra are weighted based on the spectrum quality. As such, the technique will only work if the quality of the spectrum is appropriately assessed. If the assessment of the spectrum quality is flawed, the technique will not prevent the sounding curve from becoming either a meaningless steep slope or overly smoothed.

2.2.2 Technique to remove the noise with both the residual and an unknown noise parameter

This technique separates the unknown impedance Z into two parameters, that is, the signal component Z_S and the noise component Z_N. Z_S is constrained to be continuous _in the direction of frequency and Z_N is constrained to be Gaussian-random, as shown in (8).

$$\begin{bmatrix} H & H \\ \alpha C & O \\ O & W \end{bmatrix} \cdot \begin{Bmatrix} Z_S \\ Z_N \end{Bmatrix} = \begin{Bmatrix} E \\ O \\ O \end{Bmatrix} \tag{8}$$

This technique removes the noise not only by the residual, but also by an unknown noise parameter. Data weight is assigned only to the identity matrix I, which is a constraint for the noise parameter. The degree of the constraint for the signal is continuous, hence a certain continuity of the sounding curve is maintained. The technique prevents the sounding curve from becoming meaninglessly steep slope or overly smoothed, even if the spectrum quality has been evaluated poorly. Further, the residual and the unknown noise parameter are expected to remove the noise sufficiently.

3. Synthetic test

This paper compares the two techniques using a simple synthetic calculation. For the sake of simplicity, the synthetic signal is set to a real scalar.

3.1 Technique to remove noise with the residual only

To evaluate the performance of the technique, H and E in equation (7) are given the Gaussian noise matrix H_G and vector E_G. The standard deviation of the Gaussian noises is 0.4.

$$\begin{bmatrix} w(H + H_G) \\ \alpha C \end{bmatrix} \cdot \{S\} = \begin{Bmatrix} w(E + E_G) \\ O \end{Bmatrix} \tag{9}$$

where S is the unknown signal; the nonzero component in the matrix of H is 1; Vector E includes 80 components and is assigned a sine wave with an amplitude of 2 and a period of 80; and w is a weight value assigned to each sample. All of the components are given the same weight value. Figure 1 shows the calculation results for each size with the weight w. The obtained output signal comes close to the true signal in only one case, namely, where w = 1. This means that a correct signal is unattainable without an appropriate w. If the quality of the data is underestimated (w = 0.01-0.1), the obtained curve will be too smooth. Conversely, if the quality of the data is overestimated, the curve will become meaninglessly steep. Thus, the technique to remove noise with the residual seems to require an appropriate evaluation of the data quality. This is problematic, as the data quality is often too difficult to evaluate well.

3.2 Technique to remove noise with the residual and an unknown noise parameter

As in the previous section, H and E in equation (8) are given the Gaussian noise matrix H_G and vector E_G. The standard deviation of the Gaussian noises is 0.4.

$$\begin{bmatrix} H + H_G & H + H_G \\ \alpha C & O \\ O & wI \end{bmatrix} \cdot \begin{Bmatrix} S \\ N \end{Bmatrix} = \begin{Bmatrix} E + E_G \\ O \\ O \end{Bmatrix} \tag{10}$$

Figure 2 shows the calculation results for each weight w. In the range of w = 0.1-100, the obtained output signal is close to the true signal. The appropriate range of w here is wider than that in the technique to remove noise with the residual only. As long as the data quality is not extremely undervalued (w = 0.01), the technique using both the residual and an unknown noise parameter will give a curve without excessive smoothing. If, on the other hand, the data quality is overestimated (w = 10-100), the technique will not easily give a sounding curve with a meaninglessly steep slope.

Further, the noise tends to be removed mainly by the unknown noise parameter when the data quality is underestimated (w = 0.01-0.1) and mainly by the residual when the data quality is overestimated (w = 10-100). This

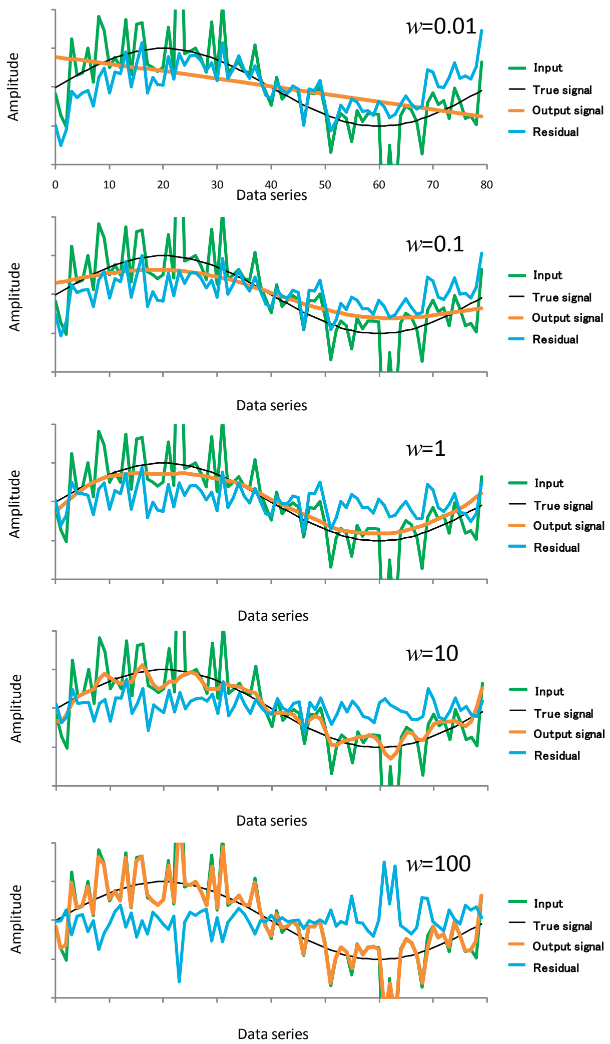

Fig. 1 The calculation results for each weight w using the technique to remove the noise with the residual.

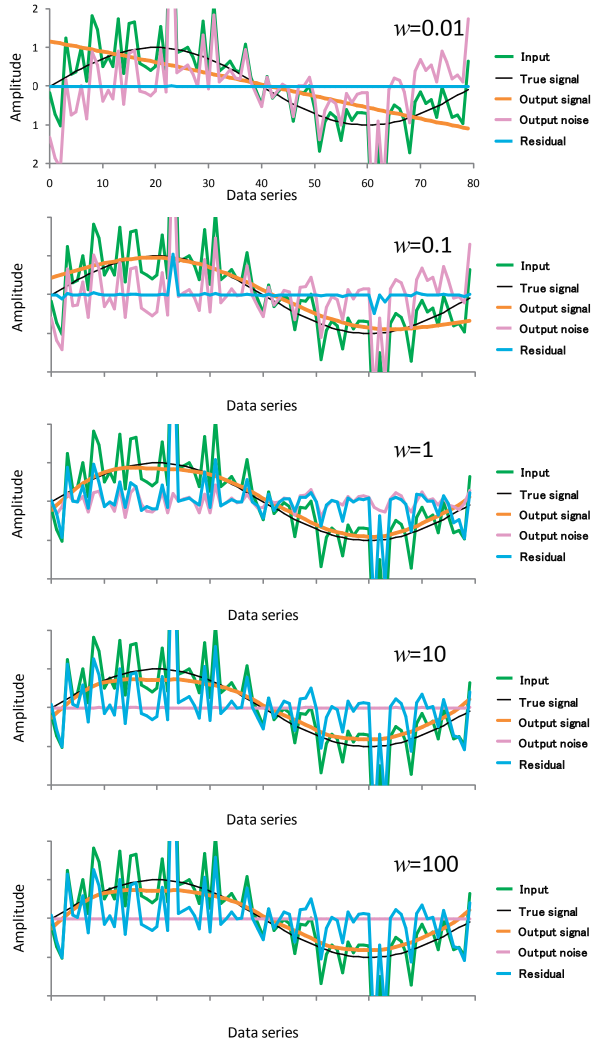

Fig. 2 The calculation results for each weight w using the technique to remove the noise with both the residual and an the unknown noise parameter.

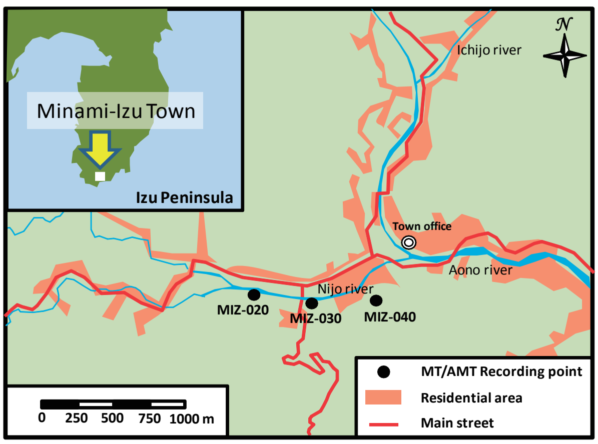

Fig. 3 Map of the recording points where the magnetotelluric data were applied.

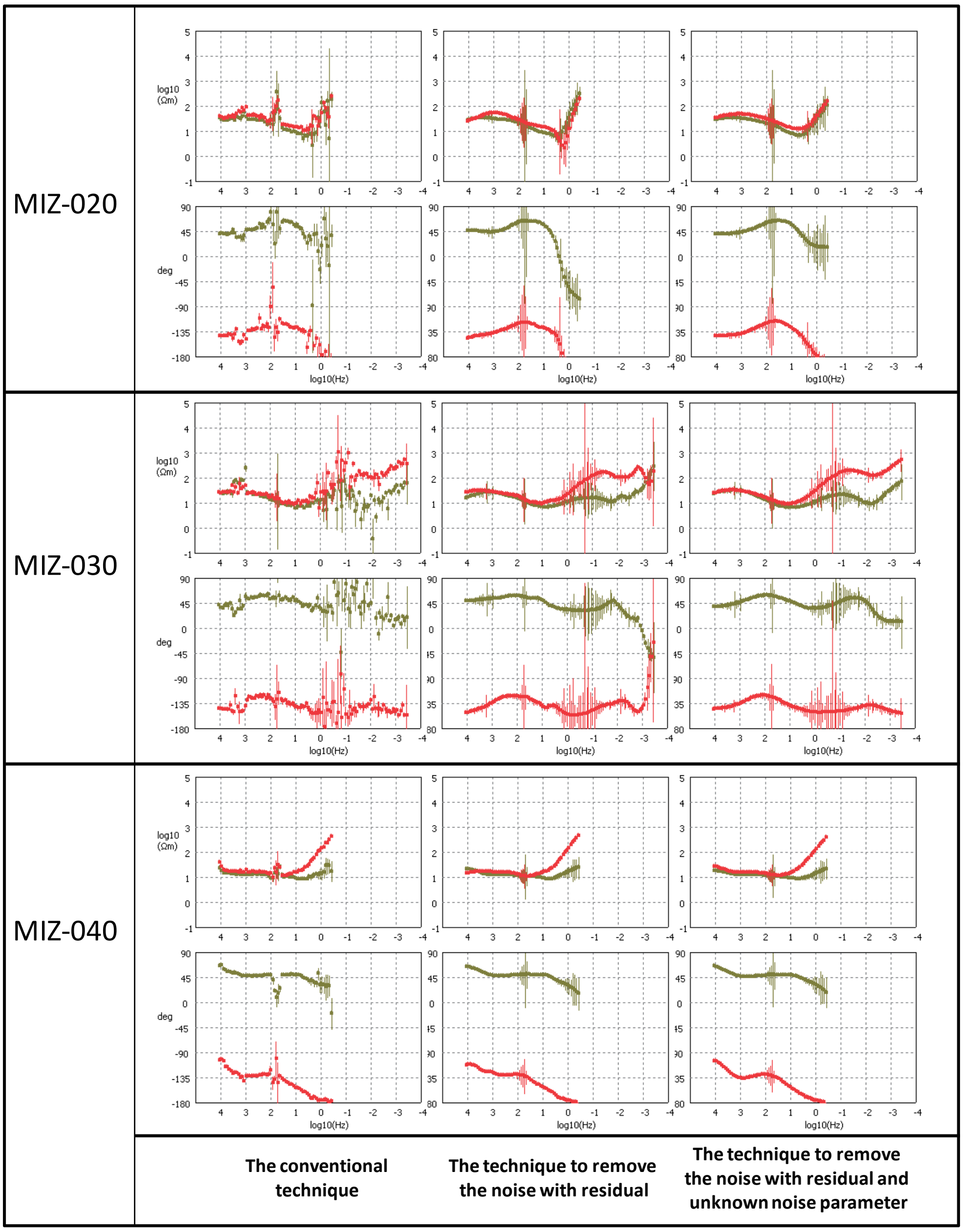

Fig. 4 Examples of the calculation results for MIZ-020, 030, and 040.

suggests that the technique using both the residual and an unknown parameter produces a good result even without an appropriate evaluation of the data quality.

4. Validation based on real data

4.1 Applied data

In the next phase of our study we tried to validate the applicability of these techniques by applying real MT and AMT (Audio frequency MT) spectrum data with strong artificial noise near an urban residential area (see Figure 3). The data were acquired in a project led by the Ministry of the Environment to promote "Core Technology for the Prevention of Global Warming 2011". The research was performed by a research team collaboratively formed by The National Institute of Advanced Industrial Science and Technology, Tokyo Electric Power Services Co., Ltd., and Nittetsu Mining Consultant Co., Ltd.

As the reference point for MT, we used a long-term continuous observation point located in the west of Waga Town, Iwate Prefecture, approximately 600 km north of the study area. The reference point for AMT was recorded in a forest about 10 km northwest of the study area.

4.2 Results

Figure 4 shows examples of the calculation results for MIZ-020, 030, and 040, three zones located within 1 km from an urban residential area. The sounding curves to the left in the figure were obtained by a conventional technique. The continuity of the curve was poor in some frequency bands at every station examined. The figure in the middle was obtained by the technique to remove noise with the residual only. Here, the continuity of the curves was improved at all of the stations. A number of steeply sloped curves remained, however, in some of the frequency bands. The figure on the right is a curve obtained by the technique to remove the noise with both the residual and an unknown noise parameter. Here we can see improved continuity of the curves in all of the stations, and no steeply sloped curves.

Thus, the technique to remove the noise with both the residual and an unknown noise parameter is unlikely to give a meaninglessly steep slope or over-smoothed sounding curve even if the evaluation of the spectrum quality is lacking or incomplete.

5. Conclusion

In some cases, magnetotelluric data collected from real-world environments have qualitative deficiencies that are impossible to improve. We have been addressing this problem by working out the robust magnetotelluric impedance based on the presence of noise in the spectrum. So far we have proposed two calculation techniques for this purpose, namely, a "technique to remove noise with the residual only" and a "technique to remove noise with the residual and an unknown noise parameter". In this study we compared the results of these techniques using both synthetic data and real data. As it turned out, the former technique was found to require appropriate evaluation of the spectrum quality. On the contrary, the latter technique calculates the robust impedance, even when the spectrum quality cannot be appropriately evaluated. We thus conclude that "technique to remove noise with the residual and an unknown noise parameter" can effectively calculate magnetotelluric impedance from a low-quality spectrum.

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